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ABSTRACT

This volume is the fifth in a series of 29 coordinated MINNEMAST units in mathematics and science for kindergarten and the primary grades. Intended for use by kindergarten teachers, this unit guide provides a summary and overview of the unit, a list of materials needed, and descriptions of four groups of activities. The purposes and procedures for each activity are discussed. Examples of questions and discussion topics are given, and in several cases ditto masters, stories for reading aloud, and other instructional materials are included in the book. This unit presents activities related to measurement. Individual sections of the unit concern: (1) length, (2) area, (3) volume, and (4) time. In each section, the activities are concerned with comparison of objects (or durations), and development of ideas related to ordering. A brief list of children's books related to measurement is included. (SD)

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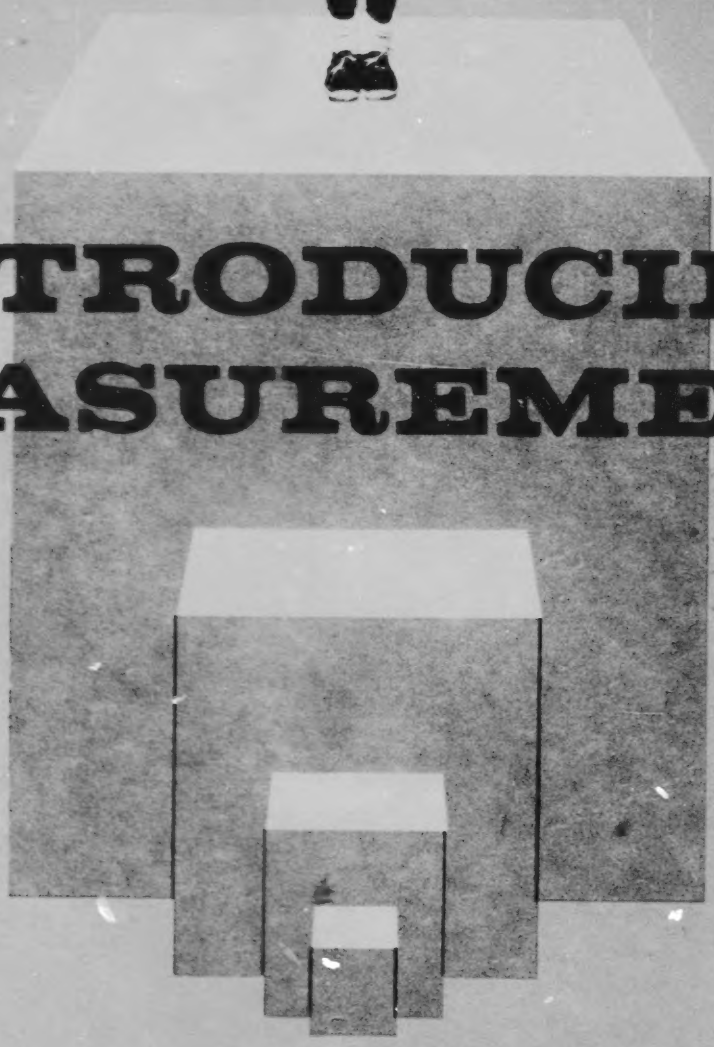

UNIT 5 -- 2

MINNEMAST

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

SE 021 210

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	7.	INTRODUCING SYMMETRY
FIRST GRADE	8.	OBSERVING PROPERTIES
	9.	NUMBERS AND COUNTING
	10.	DESCRIBING LOCATIONS
	11.	INTRODUCING ADDITION AND SUBTRACTION
	12.	MEASUREMENT WITH REFERENCE UNITS
	13.	INTERPRETATIONS OF ADDITION AND SUBTRACTION
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	27.	NUMBERS AND THEIR PROPERTIES
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UNIT 5

MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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developed these materials under a grant from the

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FIFTH PRINTING
1971

Picture Story, "Up in the Morning,"

by

Heidi Schwabacher

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MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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This unit, Introducing Measurement, was developed by the MINNEMAST staff from earlier MINNEMAST materials in the light of the experience of the many teachers who have tried the activities in the classroom. This trial edition, Unit 5 of the MINNEMAST Coordinated Mathematics-Science Series, was produced under the leadership of:

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Complete List of Materials for Unit 5

(Numbers based on class size of 30.)

total number required to teach unit	item	lessons in which item is used
50-70	sheets of construction paper to make strips of varying lengths and colors, rectangles, triangles and other shapes (can be made of flannel)	1,3,4,5,7, 12,14,15
1	flannel board	1,9,11,13, 14
15	*** sets of Minnebars	1,2,5
1	set of objects of varying lengths	2
15	* opaque bags (or lunch bags)	2
10	sets of 10 twigs or sticks for each group	3
1 each	$4\frac{1}{2}$-foot red, 5-foot yellow and $5\frac{1}{2}$-foot blue and 4-foot green lengths of yarn or cord	5,7,9
10-20	sheets of crepe paper (cut to size)	6,13
1	icicle or popsicle	6
1	Slinky toy	6
1	pencil	6
1	* length of heavy string or plastic tubing	8
5-10	cans, baby food jars, etc.	8
2	* 15-foot lengths of yarn of different colors	9
4	flannel objects, varying in size and color	9,11
30	12" x 18" drawing paper	11
30	paste	11
30	scissors	11,13,14
1	cardboard playing-boards for ordering games	12
8	** sheets of cutouts for ordering games	12
90	sheets of plain, white paper	13
30	boxes of crayons	13,14,18
1	*** set of property blocks	14
30 each	** Worksheets 1 and 2	
50	** Worksheet 3	

3	3-dimensional paper objects	15
2	large blocks of different sizes	15
2	containers of different volumes and shapes	16
1	popcorn or similar filling material	16
15	* paper trays	16,18
20	* small containers, 10 of one size, 10 of another	18
15	* 12-oz. plastic containers	16,18
6	* sets of Minnecubes	17,19
1	pitcher	18,19
3	* plasticine of at least 3 different colors	18
3	jars or bottles of different shapes	19
8	** panels of story "Up in the Morning"	20
1 doz. ea.	2 kinds of seed	23
6-8	clear, plastic cups	23
6-8	sheets of paper toweling or foam packing	23
62	sheets of plain paper	24
1	penny	24
1	* feather	24
30	pairs of objects: one object in each which flutters and one which falls quickly	24
30	* clear plastic containers, 4-oz.	25
31	* sugar cubes	25,27
15	* craft or Popsicle sticks for stirring	25,27
32	Fizzies	25,27
4-6	wind-up toys	25,26
1	piano	25,26
1	record player and records (optional)	26

*kit items as well as

**printed materials available from
Minnemath Center, 720 Washington Ave. S.E., Mpls., Minn. 55455

***available from The Judy Company,
310 North Second Street, Minneapolis, Minnesota 55401

BIBLIOGRAPHY

Beim, Jerrold	The Smallest Boy in the Class	Morrow and Co., 1949
Leodhas, Sorche	Always Room for One More	Holt, Rinehart, Winston, 1965
Liorri, Leo	Inch by Inch	Obolensky, Inc., 1960
	Swimmy	Pantheon, 1963
Ramirez, Carolyn	Let's Think About Time	Hart Co., 1965
	Small as a Raisin, Big as the World	Harvey House Inc.
Watson, Nancy	When is Tomorrow?	Knopf, 1955

INTRODUCTION

SUMMARY OF CONTENTS

The activities in this unit introduce the children to:

- Rudimentary concepts concerning the space and time properties of the child's world. These properties include length, area, volume, time duration and time order.

- Qualitative (non-numerical) comparisons of the length, area, or volume properties of objects, and the time duration and time order properties of events.

- Techniques which will enable them to order three or more objects according to their length, or area, or volume property, or events by their time duration.

- Terms such as "greater than," "less than" and "appears to be the same as" when used in the context of space and time comparisons.

COMMENTARY

This is the first of a series of units dealing with measurement. In previous kindergarten units, children have had experience with describing and classifying objects by their properties. The next step is to have them measure some of the properties of objects. Often a meaningful description of an object can be given only after some of its properties have been measured.

A measurement is generally thought of as an association of a number with some property of an object, but measurement also includes a simple comparison of the properties of objects in terms such as "greater than," "less than," or "appears to be the same as." In this unit, only this qualitative (non-numerical) measurement will be used. Actually, comparison is the basis of all measurement. In all probability, early man made very gross comparisons of the sizes of animals, lengths of time durations, and so on. It was later, after the idea of number was developed, that measurements

were made more precise by comparing with reference units and by counting such units. Here, children will make only rudimentary comparisons. Later, in Unit 12, Measurement with Reference Units, they will refine these comparisons so that the results can be recorded numerically.

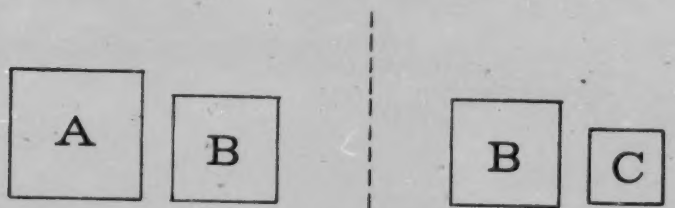
A comparison can be made between two -- and only two -- objects at a time. In other words, no matter how many objects are to be arranged in order, the procedure must be to break down the operation into numerous comparisons of two objects at a time. This comparing of objects in pairs is called binary comparison. Measurement is essentially a series of binary comparisons. For example, in measuring lengths, we either compare the objects with each other or compare the object we want to measure with a reference object such as a ruler. In measuring time durations we may be comparing two events themselves or comparing an event with a reference event such as the movement of the hands on a clock. Whenever two objects are brought together for comparison, the measurement is called direct. If a reference object is used to compare objects that cannot conveniently be brought together, the measurement is indirect.

A series of binary comparisons will enable the child to order or rank objects and events according to certain properties. The properties investigated in this unit are length, area, and volume for objects, and time duration for events. Length is considered as a measure of a line segment, area the measure of a region on a surface, volume the measure of the space occupied by (or within) an object, and duration the measure of the time an event lasts. We will also investigate the time order relation between events, which is similar to the space order relation between objects. These order concepts will be developed more fully in Unit 10, Describing Locations.

The results of comparing length, area, or volume are found to be invariant under many conditions. Invariance implies that the result of the comparison remains the same regardless of where, when, or how the comparison is repeated. There is absolutely no way of knowing whether the length of an object has changed when the object is observed in a different

location or at a different time. One can only measure the object by a binary comparison with the same reference object at both locations or times. If, by comparison, an object has greater length than the reference object at one location and time, and this same relationship exists at another location or time, the comparison can be called invariant.

An underlying principle that is emphasized throughout this unit is that of transitivity. Transitivity is illustrated in the following example. If the area of object A is greater than the area of object B, and the area of object B is greater than the area of object C, we can predict, by transitivity, that the area of A is greater than the area of C. This principle, which permits us to predict a comparison on the basis of two other comparisons, applies to length, volume and time duration comparisons, as well as to time ordering.



Comparison I Comparison II

In the activities dealing with time duration, some very simple before and after concepts of time ordering will be introduced. In this case, as probably in many others, the ideas will not be completely new to the children. The intent is to clarify their thinking and broaden their experiences.

In several lessons in this unit the need for recording arises. You will make records using the symbols for "greater than" (>) "less than" (<), and "appears to be the same as" (≐), but the children should not be required to write or read them. The children should begin to see that there are many situations in which recording is essential to an investigation. This early introduction to measurement, which precedes a more extensive treatment in a first grade unit, is intended to show that it is sometimes necessary to keep records, and that we have handy methods for doing so.

NOTES ON TEACHING THE UNIT

For a typical class, this unit should take about six weeks to complete. Each lesson should take about one day, although a few of the lessons will require two days.

Do not feel that you must complete every part of every lesson. Use your professional judgment to determine which of the many possible activities in each section are necessary for your class to attain the objectives.

Suggested questions are given throughout the manual in capital letters with possible responses in parentheses. You should not feel bound by these questions -- they are meant only as guides and often serve to clarify procedures of a lesson.

Many games and activities are included which the children may enjoy playing during their free time. Encourage them to do this. Sometimes you may wish to delay part of an activity to use as a review later in the unit.

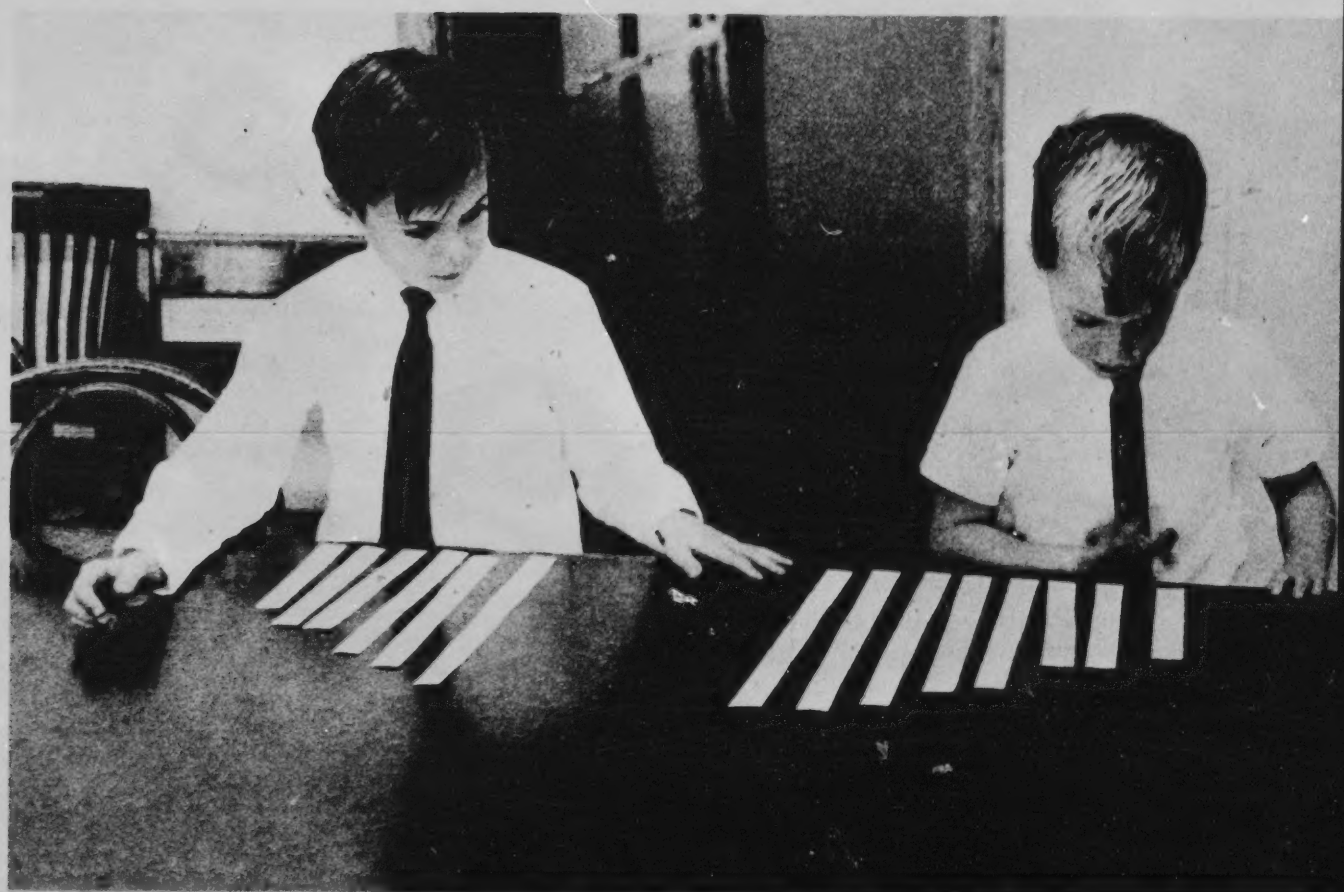
OBJECTIVES

The behavioral objectives of Introducing Measurement will be satisfied if:

- When confronted by two or more objects, the child can physically rank the objects according to their lengths, areas or volumes.
- When confronted by two events that start or end together, the child can detect which had the greater duration.
- After completing either or both of the above tasks, the child is able to express his findings orally using such terms as "greater than," "less than," and "appears to be the same as."
- The child has an understanding of the terms "length," "area," "volume," and "time duration," so that he can answer questions and carry out directions involving these terms.

- The child can predict that a comparison for a specific property of two objects will not change when the comparison is performed at a different time or at a different location. He can also make an appropriate test to evaluate his predictions.

- The child can apply the transitivity principle under appropriate conditions.



SECTION 1 COMPARING LENGTH

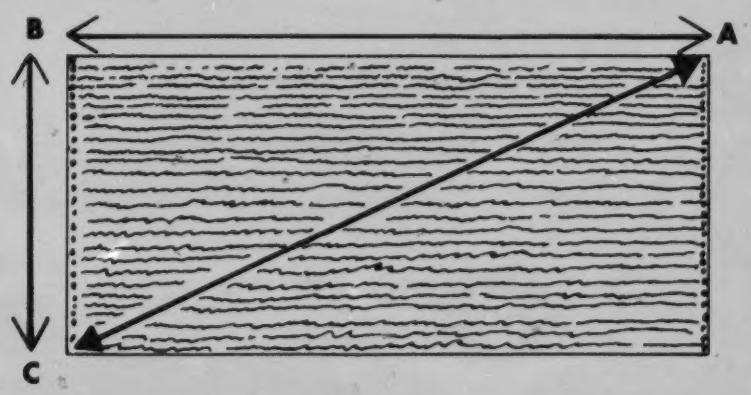
PURPOSE

The activities here are designed to have the following results:

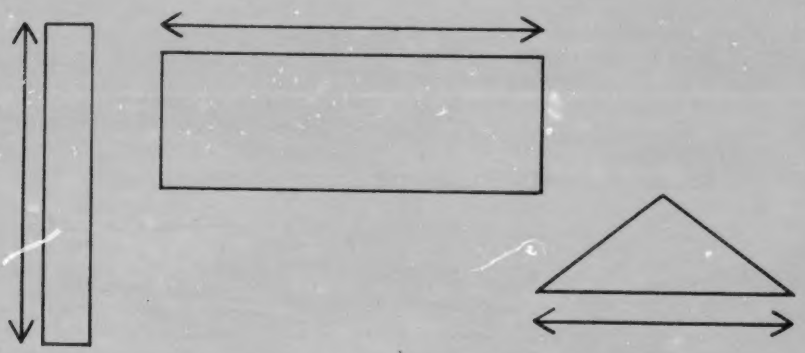
- The child learns how to make a comparison of the lengths of two objects and to express the result.
- He is able to order a set of three objects according to the property of length.
- He is able, when given a length comparison of two objects, to predict the result of a new comparison of the lengths of the same objects, based on the concept of invariance.
- He realizes that, if object A has a length greater than object B and object B has a length greater than object C, then object A has a length greater than object C (the transitivity principle).

COMMENTARY

In this section, length will be developed as a measure of a line segment and as a property of objects. In the case of a given rug one could measure the length from B to C or the length from A to C, as well as the length from A to B. This is a somewhat more general meaning than is found in common usage, in which length is often a measure of the greatest outer linear dimension of an object.



The children will first consider length in the more familiar sense, indicated by the arrows in the illustrations below, and then in the more general sense that applies to any line segment.



In kindergarten, children use many objects of different lengths (blocks, crayons, building boards, etc.). This variety in the lengths of objects in the room enables you to provide many opportunities for the children to compare the lengths of two objects.

In Lesson 1 of Section 1, children use sets of Minnebars for the first time. These are wooden bars of twelve different lengths. Each bar can be identified by its color, length and the number of units marked on one surface. Each set of Minnebars should contain:

13 one-unit bars
7 two-unit bars
4 three-unit bars
3 four-unit bars
3 five-unit bars
2 each of lengths six through ten
1 each of lengths eleven and twelve

Where Minnebars are used in this unit, the focus is on the overall length of the bars and not on the number of units. The various colors of the bars help the children pick out the different lengths and make it easier to identify them in discussions.

When working with Minnebars or any other objects, use the terms "greater length" and "lesser length" or "longer than" and "shorter than" frequently, and give the children many opportunities to use them. For instance, you might ask a group building roads with blocks to compare the lengths of the roads and tell you about them. Or children might be encouraged to tell you that some of their crayons are "longer than" others.

If children are often permitted to use natural objects as part of the lessons dealing with length, they will have more learning experiences and the lessons will be more interesting. They could compare the lengths of two twigs or arrange three shells in order by length. They might also enjoy comparing the lengths of their shadows.

This kind of incidental learning should go on throughout the year. The lessons provide a suggested structure, but there is no limit to other activities you might devise to achieve the objectives.

Lesson 1: INTRODUCING LENGTH

In this lesson the children will begin to consider the length of the greatest obvious dimension of an object. They will compare objects two at a time and tell which has the greater or lesser length. In some of the situations they will not be able to see a difference -- the objects will appear to be the same length.

The children will use Minnebars in this lesson for the first time. They should be allowed to play with these during their free time before and after the unit activities.

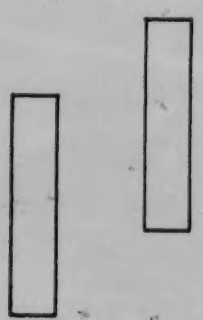
Several activities are included. Use your own judgment as to which to use for your class. Feel free to devise similar activities using different materials or to delay using some of the activities included here until later in the unit.

MATERIALS

- heavy paper (of 4 or 5 different colors) cut into narrow strips 2" to 12" long (enough strips so that each child in your class may have two)
- flannel board and strips
- Minnebars

PROCEDURE

[A] Place two strips of the same length on the flannel board so that the ends are on different levels.



WHICH IS LONGER?

WHICH IS SHORTER?

HOW CAN YOU BE SURE?

HOW CAN WE FIND OUT WHICH HAS GREATER LENGTH?

Have a volunteer show a method of comparing.

The children should realize that to show that the objects apparently have the same length, both pairs of ends must line up with nothing left over.

B Divide the class into groups of four. Give each group eight of the strips of paper described in the material list. Ask each child to pick out two of the strips from his group's set and to describe and compare his two strips.

WHICH OF YOUR STRIPS IS LONGER? HOLD IT UP.

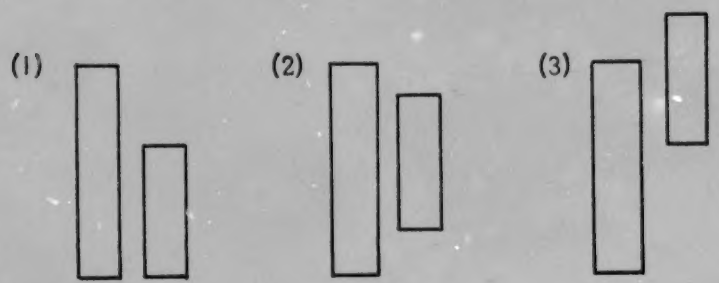
FIND A WAY TO SHOW WHICH OF YOUR STRIPS IS LONGER.

Now have everyone in each group put his first pair of strips back in the group's pile and choose two different ones.

WHICH OF YOUR NEW STRIPS IS LONGER? HOLD IT UP.

CAN SOMEONE SHOW US ONE WAY OF TELLING WHICH OF HIS STRIPS IS LONGER?

Have several children show their methods of comparison. Try to have all of the following methods shown. Method 3 is not a decisive test.



C Ask the children to find two objects in the room that appear to have different lengths. Have them tell which is the longer and test their decision by making the comparison.

D Give one set of Minnebars to each pair of children. One child should choose two Minnebars and ask the other child which bar has the greater length. They should take turns telling and check by actually making the comparison.

E Game: Giant or Dwarf

Have many strips of heavy paper of different lengths prepared. Minnebars might also be used here. Select two children to demonstrate the game. Give each child six strips (or Minnebars) of different lengths and ask that he keep them out of his partner's sight. At a signal, each child secretly chooses one of his strips. The partners then compare the two strips they have chosen. The child with the longer strip wins both strips and places them in a separate pile. If the strips are of similar lengths, each keeps his own strip. The game continues through six matchings. The child with the most strips at the end is the winner. After the demonstration, the entire class should have opportunities to play in groups of two. For variation, the winner could be the one with fewer strips.

Lesson 2: USING REFERENCE OBJECTS IN COMPARING LENGTHS

In this lesson, the children will compare one object with other objects, and become more familiar with describing an object in comparative terms. They will see, for instance, that a pencil has greater length than a crayon but less length than a yardstick. Emphasize such comparative descriptions in all measurement discussions throughout this unit and the rest of the year. Also use comparison terms for properties other than length:

IS THIS BLOCK HARDER THAN THIS BALL OF COTTON?

IS THIS SHEET OF GREEN PAPER DARKER THAN THAT ONE?

Now the children should begin to see that all comparisons are relative -- the results depend on the objects compared.

MATERIALS

- a set of 10-15 objects of varying lengths -- pencils, blocks, crayons, books, soda straw, etc.
- Minnebars
- opaque bags (paper lunch bags would do)

PROCEDURE

[A] Place the set of objects where all the children can see them. Hold up one object (e.g., a pencil).

FIND AN OBJECT WHICH IS LONGER THAN THE PENCIL.

FIND AN OBJECT WHICH IS SHORTER THAN THE PENCIL.

SHOW ME HOW YOU TEST WHICH IS LONGER.

(When they see the comparative aspect, ask similar questions without holding up the object.)

FIND AN OBJECT WHICH IS SHORTER THAN THE BOOK.

WHO CAN FIND AN OBJECT WHICH HAS GREATER LENGTH THAN THE CRAYON BUT LESS LENGTH THAN THE STRAW?

B Game: Grab-Bag Matching

This is played by two children. Give each pair an opaque bag of assorted Minnebars. Without looking in the bag, each child draws one bar to serve as his reference object. The child with the longer reference object collects bars that are longer than his reference object and the other child collects those shorter than his reference bar.

The child with the longer reference bar begins the play by drawing another bar from the bag and comparing it with his reference. If it is longer than the reference he keeps it. If it is the same length or shorter than the reference, he puts it aside (not back in the bag). The other child then makes his draw and retains the bar only if it is shorter than his reference. Each takes his turn until all the bars are drawn. The child having the greater number of bars is the winner.

As children play the game, some will come to realize that selecting the reference and other bars by touch gives an advantage.

Lesson 3: CLASSIFYING OBJECTS BY LENGTH

In this lesson the children describe and classify objects according to the property, length. They will sort a set of strips of various lengths using a reference object. To illustrate the procedure, start the lesson with the flannel board and flannel objects.

MATERIALS

- 10 paper strips (from Lesson 1) of varying length (2" to 12") for each group of 3 or 4
- set of 10 twigs or sticks for each group, collected before the lesson

PROCEDURE

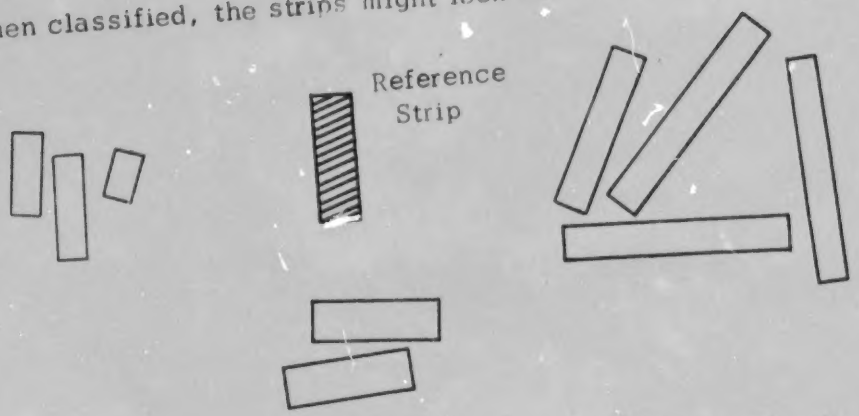
[A] Give each group of three or four children one set of ten paper strips or have them take out their set of sticks. One member of each group should place one of the strips or sticks from the mixed pile in the center of his group's working area.

Tell the class that each group should use this first strip (or stick) as their reference strip. Tell the children they should compare all their other strips, one at a time, with this reference strip.

Then have them make subsets. On one side of the reference strip, they are to make a subset of all the strips which are shorter than the reference object. On the other side, they should make a subset of all strips which are longer than the reference strip. Strips that seem to have the same length as the reference strip should be put in a subset under it.

Repeat this activity, permitting another child from each group to select the reference strip.

When classified, the strips might look like this:



With each group, emphasize the fact that two (or three) subsets have been made -- one subset of objects greater in length than the reference object, one subset of objects less in length than the reference object, and perhaps a third subset of objects which seem to have the same length as the reference object.

After the children have classified their strips you might give them pieces of yarn to put around each subset they have formed.

B (If you feel the children may be sensitive about the following game, omit it or use handspan.) Choose one child of medium height as a reference child. Ask several of the other children to compare their heights (lengths) to that of the reference child. (You may need a referee!) Form one subset of children with greater length (height) on one side of the reference child, another subset of shorter children on the other side and possibly a third subset of children of the same length as the reference child.

Repeat, using yourself as the reference object. Ask what a subset of children taller than, or a subset the same height as, the reference object in this case is called. (The empty set.)

Repeat the activity, using a chair -- or some other object -- that has a lesser length than any of the children. The purpose here is to add to the children's understanding of the fact that all comparative descriptions of an object depend on the object with which the comparison is made.

C The following riddles are intended to provide practice with identifying objects by various properties, including length.

Hold up a pencil.

I'M THINKING OF SOMETHING THAT IS LONGER THAN THIS PENCIL.

IT IS WOOD.

IT IS FLAT.

IT IS IN THE SET OF BLOCKS.

WHAT AM I THINKING OF? (A board from the blocks.)

HOW CAN YOU SHOW THAT IT IS LONGER THAN THE PENCIL? (A child gets the board and compares it with the pencil.)

Stretch a 12" piece of yarn taut.

I'M THINKING OF SOMETHING THAT IS SHORTER THAN THIS PIECE OF YARN.

IT IS RED.

I CAN COLOR WITH IT.

WHAT AM I THINKING OF? (A red crayon.)

HOW CAN YOU SHOW THAT IT IS SHORTER THAN THE YARN? (A child gets a red crayon and compares it with the piece of yarn.)

Hold up scissors.

I'M THINKING OF SOMETHING THAT IS ABOUT THE SAME LENGTH AS THESE SCISSORS.

IT IS DUSTY AND GRAY.

IT IS HARD ON ONE SIDE.

IT IS SOFT ON THE OTHER SIDE.

WE USE IT TO WIPE THE CHALK MARKS OFF THE CHALKBOARD.

WHAT AM I THINKING OF? (A chalkboard eraser.)

HOW CAN YOU SHOW THAT IT IS ABOUT THE SAME LENGTH AS THE SCISSORS? (A child gets an eraser and compares its length with that of the scissors.)

After the children have had a few experiences with these or similar riddles, they can make up other riddles.

Lesson 4: INVARIANCE OF LENGTH COMPARISONS

This lesson deals with the idea that length comparisons are invariant under certain changing conditions. These changing conditions might involve moving the objects to a new location or orientation, or comparing them at another time. The children should see that throughout the activity a comparison can only be made when the objects are lined up.

MATERIALS

- 8 to 10 strips of flannel or heavy paper -- some of these should be of the same length and all of them should be of different colors.
- resting mat

PROCEDURE

[A] Take two strips of the same length and arrange them horizontally on the flannel board.

WHICH OBJECT HAS THE GREATER LENGTH? (They are the same length.)

WILL THEY STILL BE THE SAME LENGTH IF I ARRANGE THEM UP AND DOWN ON THE BOARD?

After the children have made their predictions, ask them to suggest ways of checking them. Have them test these predictions by making the vertical comparison. Use the following questions to introduce similar predictions and checking situations.

CAN YOU PREDICT WHICH STRIP WILL HAVE THE GREATER LENGTH IF WE PLACE THEM BOTH ON THE FLANNEL BOARD SIDE BY SIDE, BUT KITTY-CORNERED?

CAN YOU PREDICT WHICH STRIP WILL HAVE THE GREATER LENGTH IF WE COMPARE THEM FLAT ON THE FLOOR? DIAGONALLY IN THE AIR?

Repeat the above activity -- predicting and checking -- with strips of unequal lengths. In this situation, the children should notice through different comparisons of the same strips that the orientations of the strips during the comparison operation do not make a difference in the result.

Place two strips of about the same length on the flannel board.

CAN YOU BE SURE ABOUT HOW THE LENGTHS OF THESE STRIPS COMPARE IF THEY ARE PLACED LIKE THIS? (No, not if they are about the same length.)

HOW SHOULD WE COMPARE THEM? (Place them side by side.)

Have the children make the comparison.

B Use some of the comparisons above and ask the children whether the result of the comparisons would be the same the next day. Ask them to predict what they would expect to find if they made the same comparisons the next day.

HOW CAN WE REMEMBER WHICH HAS THE GREATER LENGTH?

I'M GOING TO PUT THE RECORD ON THE BLACKBOARD THIS WAY:

LENGTH (In red chalk) < LENGTH (In yellow chalk)

IT MEANS, "THE LENGTH OF THE RED STRIP IS LESS THAN THE LENGTH OF THE YELLOW STRIP."

IF THEY ARE ABOUT THE SAME, I WILL MAKE A RECORD LIKE THIS:

LENGTH (In red chalk) $\doteq$ LENGTH (In yellow chalk)

THIS MEANS, "THE LENGTH OF THE RED STRIP APPEARS TO BE THE SAME AS THE LENGTH OF THE BLUE STRIP."

Many children will be capable of learning the word "length" as a sight word and can associate the color of the word with the appropriate object, even though they may not be able to read the whole phrase. You could read their record to them as, "The length of the red object is less than the length of the yellow object." (You may wish to point out to the class that the larger end of the inequality symbol -- < -- is toward the greater length.) The major purpose here is to motivate the need for keeping records. Leave such a record on the board and have the children repeat the comparison the next day to check whether the results of their comparison remain the same from one day to the next.

C Ask a child to lie on a resting mat.

WHICH HAS THE GREATER LENGTH -- MARY OR HER MAT?

Now ask the child to lie somewhere away from the mat.

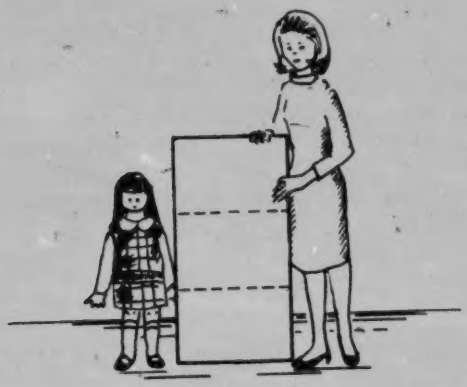
WHICH HAS THE GREATER LENGTH NOW?

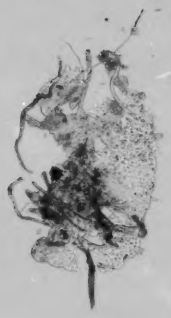
After they have predicted, ask another child to place the mat beside the child to check the prediction.

COULD WE MAKE THIS COMPARISON GOING UP AND DOWN TOO?

WOULD OUR RESULTS STILL BE THE SAME?

Try this method by holding the mat up beside the child. Be sure one end of the mat is aligned with the feet of the child.





Lesson 5: ORDERING BY LENGTH

Here the children compare and order three objects by length. In this lesson, they will measure lengths of non-rectangular, as well as rectangular, objects.

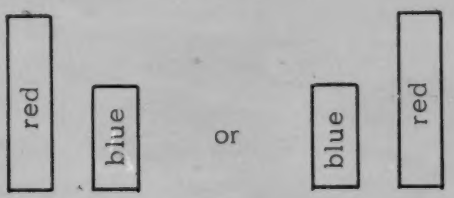
MATERIALS

- narrow strips of paper or flannel of different colors
- 3 pieces of yarn (or cords marked by yarn) identified as follows: 4 1/2' - red, 5' - yellow, 5 1/2' - blue (The extra colors of yarn may be obtained from the first grade kit.)
- Minnebars
- several triangular shapes of flannel or heavy paper of different colors

PROCEDURE

A Show the children two strips of paper or flannel (e.g., red and blue).

WHICH STRIP IS LONGER?

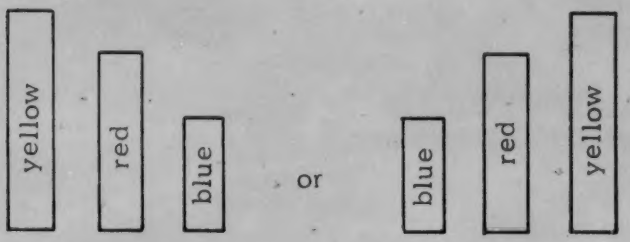


Show them a third strip (e.g. yellow).

IS THE YELLOW STRIP LONGER THAN THE BLUE?
(Children check.)

IS THE YELLOW STRIP LONGER THAN THE RED?
(Children check.)

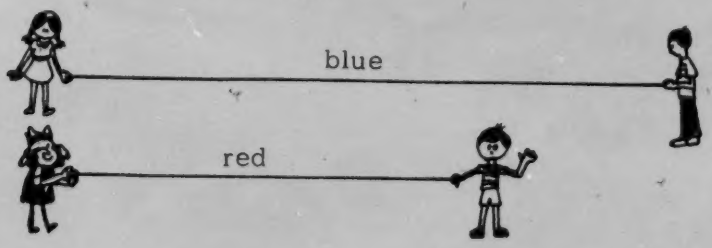
WHERE COULD WE PLACE THE YELLOW STRIP TO SHOW THAT IT IS LONGER THAN BOTH THE BLUE AND THE RED STRIPS? (Children should suggest the two arrangements.)



B Lay three pieces of coiled yarn (yellow, blue, red) -- or cord marked with yarn -- on the floor in no particular order.

COULD WE ORDER THESE CORDS BY THEIR LENGTHS? HOW?

Guide the children to suggest some method of comparing the length of two of the cords by stretching them out with two ends lined up. Have the longest and shortest (red and blue) compared first.



Have the children agree on how to place the cords in order. If they decide to put the shortest cord on the left, make a record on the chalkboard in colored chalk, or on chart paper with crayons, using the "less than" symbol. But, to introduce the "greater than" symbol (>), suggest -- if the children do not -- that the cords can also be arranged in a different order, with the longest at the left. Record this on the board or chart:

LENGTH (blue) > LENGTH (red)

THIS MEANS THE LENGTH OF THE BLUE CORD IS GREATER THAN THE LENGTH OF THE RED CORD.

Coil the cords up again.

NOW WE HAVE PLACED THE BLUE AND THE RED CORDS IN ORDER ACCORDING TO THEIR LENGTHS. HOW CAN WE FIND OUT WHERE TO PLACE THE YELLOW CORD?

The children should compare the yellow cord with the blue cord in the same way they compared the red with the blue. Record this new comparison for them:

LENGTH (blue) > LENGTH (yellow)

COULD WE PUT THE YELLOW AND RED AND BLUE CORDS IN ORDER NOW ACCORDING TO THEIR LENGTHS?

If the children say "No," ask why not. Presumably it will be because they know only that the yellow cord is shorter than the blue and do not know how the length of the yellow compares to the length of the red cord.

If they say "Yes," ask someone to place the cords in order.

HOW DID YOU KNOW WHERE TO PLACE THE YELLOW CORD?

DO YOU KNOW WHETHER THE YELLOW CORD IS LONGER OR SHORTER THAN THE RED CORD?

Have the comparison made and record it.

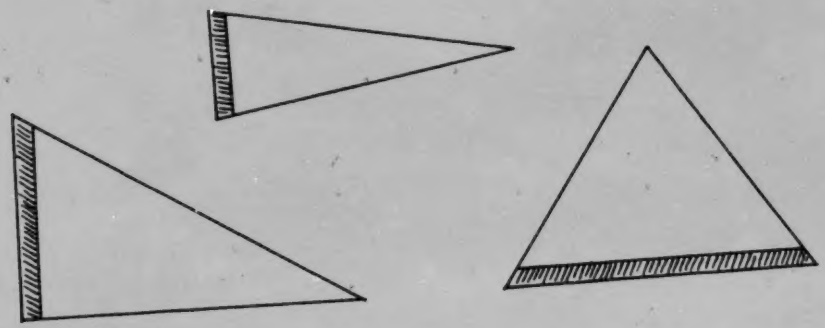
LENGTH (yellow) > LENGTH (red)

The three cords should then be arranged in order and a record made.

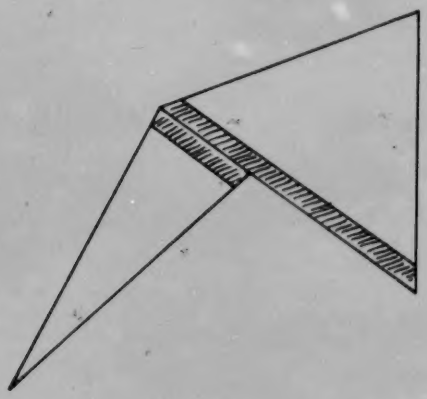
Remind the children of their record by reading it to them as they make sure the cords are arranged in the correct order. Save the record of this measurement to use in Lesson 7.

C Give the children sets of Minnebars and ask them to order any three of them according to length, making binary comparisons and lining them up. Some children may be able to order more than three at a time. If they do this, they will probably make the necessary binary comparisons by sight, which is convenient. If and when they do this, occasionally ask questions informally bringing out the fact that they are making binary comparisons by sight, without actually bringing the objects together.

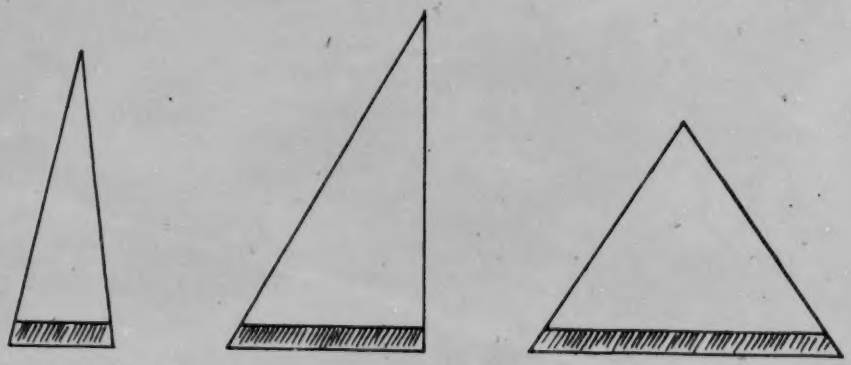
D Place several triangular shapes of different sizes on the flannel board, and mark one edge of each with crayon.



Ask a child to choose two of the triangles and compare the lengths of the marked edges. Repeat this with other pairs.



The children might want to compare pairs of unmarked edges or order all the triangles by the length of the marked edges.



Lesson 6: COMPARING LENGTHS THAT VARY

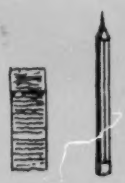
In earlier lessons of this unit the children have seen various conditions under which length comparisons are invariant. It is desirable for the children to see that length comparisons do not always remain invariant under certain changing conditions. This activity should demonstrate that there are certain materials and/or conditions for which the length comparison is not invariant. Crepe paper is used in this illustrative lesson, but you may use any of the other listed materials in a similar manner.

MATERIALS

- strip of crepe paper and a pencil
- icicle or Popsicle and a pencil
- Slinky toy and a pencil

PROCEDURE

Cut a strip of crepe paper shorter than a certain pencil.

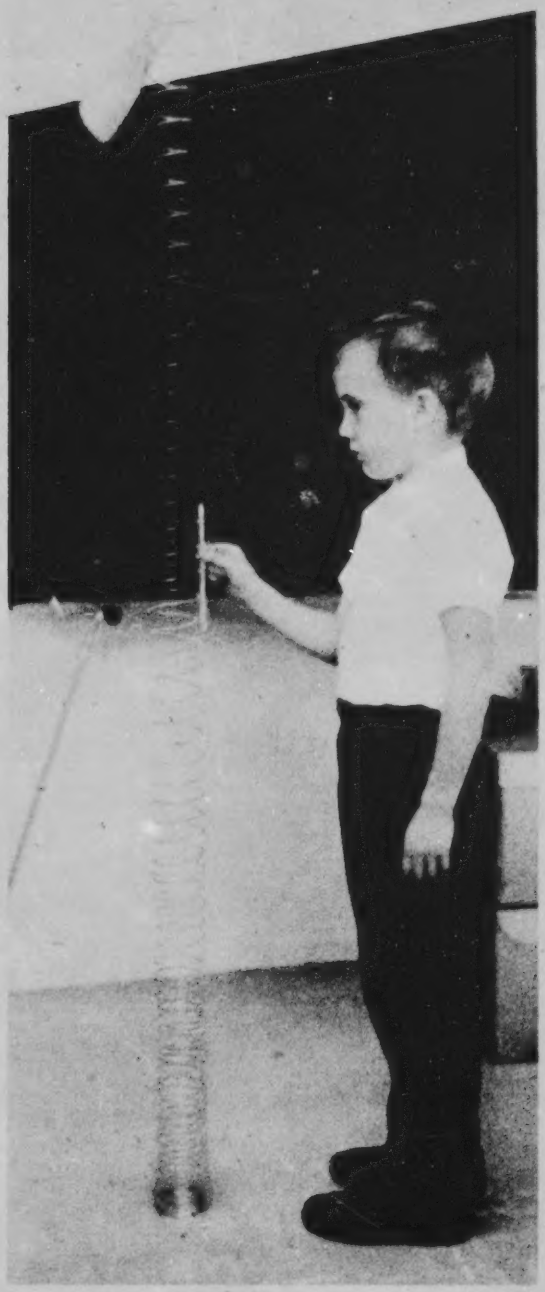


Show the children the objects and ask which object has the greater length. Record with pictures of the two objects. Then stretch the crepe paper and ask again which is longer.

SOMETIMES THERE ARE THINGS WE CAN DO TO CHANGE THE WAY TWO OBJECTS COMPARE IN LENGTH.

The icicle-pencil combination is particularly interesting. Children compare the lengths of an icicle and a shorter pencil and you make a record for them with pictures of the objects. Then they are compared again later. The children discover that some length relations are not invariant with time.

Comparing a Slinky with a pencil when the Slinky is not expanded and again when it is expanded, dramatically shows the change in relative length.



Lesson 7: TRANSITIVITY IN LENGTH COMPARISONS

In the following lesson, the idea of the transitivity of length comparisons will be introduced, using various objects. Transitivity is an idea that many of your children may recognize but be unable to verbalize. For this reason, don't require explanations in these activities.

Before teaching this lesson, you might wish to read again the paragraph concerning transitivity in the introduction to the unit, page 3. You will need to keep in mind, especially in these introductory experiences, that one can employ transitivity only under certain conditions. Given three objects A, B, and C, and knowing that B < C and that A < C, you could not infer anything about A and B. If, given A, B and C, knowing that A < C and B < A, you could infer, by transitivity, that B < C, and order the three: B < A < C. These first formal experiences with transitivity are necessarily somewhat contrived. But later on, you can emphasize transitivity more informally whenever it applies in the ordering of several objects by some property. For instance, where children have arranged several Minnebars in order by length, point out (or have them discuss) the fact that if the red bar is shorter than the yellow bar and the yellow is shorter than the blue, then the red bar is shorter than the blue bar. In all such activities -- and wherever possible -- predicting and checking should be important parts of the children's work from the beginning.

MATERIALS

- 3 paper strips of unequal lengths and of 3 different colors (longest - blue, next longest - yellow, shortest - red)
- 3 pieces of colored yarn or cord: 4 1/2' - red; 5' - yellow, from Lesson 5; 4' - green
- (optional) 3 paper strips as above, of other colors such as white, green, purple

PROCEDURE

A Show the children the red and yellow strips and ask them to compare their lengths.

WHICH STRIP IS LONGER, THE YELLOW OR THE RED? (The yellow.)

Remove the red strip from sight and show them the blue strip.

WHICH STRIP IS LONGER, THE YELLOW OR THE BLUE? (The blue.)

CAN YOU PREDICT WHICH IS LONGER -- THE RED OR THE BLUE?

HOW DO YOU KNOW?

Have the children check by an actual comparison. Some may be able to say that since they know the yellow strip is longer than the red strip, and that the blue strip is longer than the yellow strip, they can tell without comparing -- an example of transitivity.

B Put out the red, green, and yellow cords in no particular order, as you did the blue, yellow and red cords in Lesson 5, Activity B.

yellow green red

CAN YOU ARRANGE THESE CORDS IN ORDER OF LENGTH?

DOES ANYONE REMEMBER HOW TWO OF THESE CORDS COMPARED BEFORE?

Bring out the record and read it to them, with whatever help they may be able to give. Some may want to check the record but it is permissible for them to order the two cords from the results of their previous comparison, agreeing again on where to place the longer cord.

yellow > red

COULD SOMEONE PLACE THE GREEN CORD IN THIS ORDER? (No.)

WHAT ELSE DO WE NEED TO KNOW? (How the green cord compares in length with the red.)

Have the green cord compared to the red and the result recorded.

RED > GREEN

COULD SOMEONE PLACE THE GREEN CORD IN ORDER NOW? (Yes.)

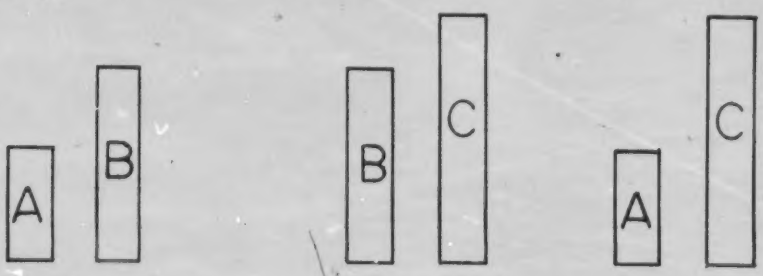
DO WE NEED TO MAKE ANY MORE COMPARISONS? (No.)

WHAT COMPARISON HAVE WE SKIPPED? (The green and the yellow.)

WHY DON'T WE NEED TO MAKE THAT ONE? (We know that the green is shorter than the red and the red is shorter than the yellow so the green must be shorter than the yellow, too.)

WILL SOMEONE CHECK WHETHER WE WERE RIGHT? IS THE GREEN CORD ALSO SHORTER THAN THE YELLOW?

C (Optional) Separate the children into three groups. Have three objects (e.g., three colored strips of paper) available. (The paper strips should be of colors other than the ones used in Activities B and C.) Do not let the children see all three objects at once. The objects are referred to here as A, B, and C:



Comparison I Comparison II Comparison III

Tell the children that each group will solve part of a problem. Give the first group strips A and B and ask them to compare their lengths and report the results. Put strip A out of sight and give the second group B and C to compare and report results. Put all three strips out of sight and have the results reported once again. Then ask if the third group can predict which will be longer -- A or C. Have them check their prediction. Some children may be able to tell why they could predict the result of the third comparison (that it was because they knew the results of the first two), but it is not expected that many will be able to express this.

Lesson 8: INDIRECT COMPARISON (OPTIONAL)

Up to this point the children have only compared the lengths of objects which they could bring together to make a direct comparison. The activities in this lesson require indirect comparison because the objects cannot easily be brought together. Therefore some intermediate object, such as string, must be compared to each of the original objects.

Use the following activities only if you feel that the interest and capability of the children are such that the lesson will be beneficial in developing an understanding of the idea that they can use something like string or parts of the body (handspan, arm length) to represent the length of an object they cannot move. Or you might wish to use these activities sometime later in the year, as review activities.

MATERIALS

- heavy string or plastic tubing
- cans, baby food jars, etc.

PROCEDURE

[A] Choose tables or other objects of different lengths in the room and ask the children to find a way to determine which is the longest.

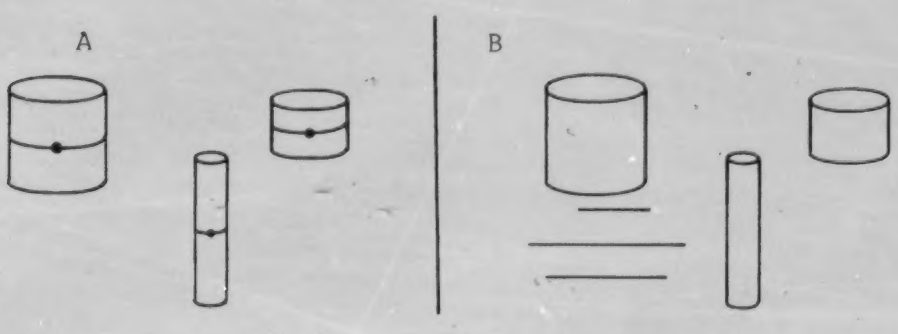
Point out that just looking at the lengths does not give an accurate determination and that such a comparison should always be checked by a better test. Ask the children to pretend that the two tables cannot be moved.

HOW CAN WE FIND OUT WHICH TABLE IS LONGER?

The children's discussion of this may bring out some unusual methods of indirect measurement. They may want to use various parts of their bodies (handspan, arm length, etc.).

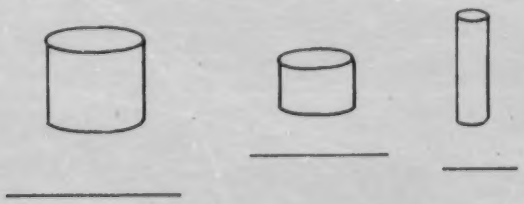
(In one trial class a child used a chair!) A convenient method might be using a string to measure one table and then comparing that length of string to the other table, as in the illustration. But let the children use any practical, appropriate technique for comparing various objects.

B Collect empty cans or cylindrical jars of various sizes. Cut lengths of heavy string or plastic tubing to match the circumference of each can, (A).



Place both the cans and the cords on a table, (B). Have a child select a cord which he thinks best fits around a can. Have him check his choice by measuring around the can with the cord. Let the other children match cords with cans.

Some children might like to arrange this set of cords in order from longest to shortest or shortest to longest. From this arrangement, let the class predict the order of the lengths around the cans. Ask how the children could check their prediction. Then have them match the cords to the cans.



SECTION 2 COMPARING AREA

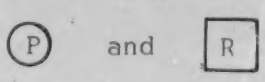
PURPOSE

This section contains activities designed to develop the children's ability to:

- Recognize area as the measure of a region on a surface.
- Estimate which of two regions has the greater area.
- Apply tests for comparing areas when they are unable to estimate a difference.
- Order three or more similarly shaped regions by area.

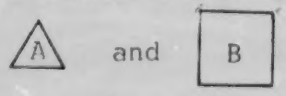
COMMENTARY

This section introduces the children to the concept of area in a qualitative (non-numerical) sense. Through the activities, the children should gain an idea of area, discover methods for area comparisons and learn to order a set of regions by area. For instance, when presented with flat regions:



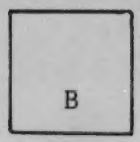
they should be able to use a test to show that region R has greater area than region P. No numerical values will be involved in these comparisons.

Area is developed as the size of a region on a surface bounded by a simple closed curve. The children will probably refer to area as the amount of "room" on a surface. The areas of two regions can be compared by determining whether one region can cover another or not. Consider regions A and B:



Since region A could be placed entirely on region B with surface still uncovered on region B, region B has greater area. This method is called a superposition test. The children will probably say, "A fits on B with room left over."

Region A



Region B

A superposed on B shows that Region A has less area than Region B, since B is not completely covered by A.

The children compare the areas of regions and discover there are several operations which do not change the areas and therefore do not change the area relations. For example, the results of an area comparison do not change even if the children move the regions to a different location, fold or cut one of the regions, or wait until the next day to make another comparison of the same regions. These situations involve the invariance of the areas and consequently the invariance of measurement comparisons. Here, as in the sections dealing with measurement of length, volume and time duration, the idea of invariance plays an important part. As the children gain experience with these special properties of objects, they should begin to realize that the results of a comparison do not vary under certain changing conditions. It would be impossible to keep and use records, or to rely on the transitive property of some relations without knowing which changes in the conditions of the comparison leave a relation unchanged.

The children will be faced with situations in which the superposition test cannot be used. Knowing that cutting a region into pieces does not change the total area of that region, they estimate which region has less area, cut that up, and fit the pieces on the other region to determine the area relation.

Use your own judgment about the amount of time to spend on any one lesson. If you find that the children understand the concepts being presented before you have finished all parts of a lesson, feel free to skip over the remaining activities.

Lesson 9: INTRODUCING AREA

Area will be considered in this unit as the "size of a region that is on a surface and is bounded by a closed curve." We do not expect the child to express his idea of area in these words but he should develop an awareness of this property of objects through the activities. Verbal skill is not required.

This lesson uses concepts of open and closed curves, inside and outside regions, and boundaries. Some children may need more review of these ideas from Unit 2, Curves and Shapes, than this lesson provides.

If they already seem to have a good understanding of area, it may not be necessary to do all the activities given here.

MATERIALS

- 2 fifteen-foot lengths of yarn (different colors) and a few shorter pieces
- flannel board and flannel objects
- 1 piece of paper

PROCEDURE

Have the children check to determine whether the two long pieces of yarn are about the same length. Then use the yarn to lay out two closed curves on the floor. Make one nearly circular and the other long and narrow. After a brief review of the fact that each curve is closed and that each has an inside region, an outside region and a boundary, ask:

HOW MANY CHILDREN COULD STAND INSIDE OF THE CIRCULAR CURVE?

HOW COULD WE FIND OUT? (We could stand inside it.)

When as many children as possible are standing on the inside region of the circular curve, repeat the questions for the other curve and have another group stand on its inside region.

COULD MORE CHILDREN STAND ON THE INSIDE REGION OF THE CIRCULAR CURVE THAN ON THAT OF THE OTHER CURVE?

HOW COULD WE FIND OUT? (By pairing members of the two sets.)

This pairing gives the children a review of one-to-one correspondence.

INSIDE WHICH CURVE COULD MORE CHILDREN STAND? (Inside the circular curve.)

WHY COULD MORE CHILDREN STAND ON THE REGION INSIDE THAT CURVE? (There was more room in it. The inside was larger.)

WHEN YOU THINK ABOUT HOW MUCH ROOM THERE IS IN A REGION, YOU ARE THINKING ABOUT AREA.

Don't insist that the children use "area" if they prefer "room" or some other term, but alternate using their word and "area" to give the meaning of area. In any case, be sure that the children use their word to refer to area, not length or volume.

Change the shape of one of the simple closed curves of yarn a number of times, asking questions like the following to familiarize the children with the idea of area:

IS THE CURVE CLOSED? (Yes.)

DOES IT ENCLOSE A REGION? (Yes.)

WHERE IS THIS REGION? (Inside the curve.)

DOES THIS REGION HAVE AREA? (Yes.)

DOES IT HAVE MORE AREA THAN THE OTHER CURVE? Let the children check by standing inside and then pairing.

Have the children change the curve to various other shapes, and repeat the questions. If no one accidentally forms an

open curve, do so yourself -- asking only if the curve is closed (no) and if it has an inside (no). If they think it does have an inside, ask several children to show where they believe the boundary is. Point out that each person could be thinking about a different boundary. You might ask several children to close the curve with a chalk line to show that each child could close it in a different way. Encourage the youngsters to make their chalk lines of various lengths.

Draw a few closed and open curves on the chalkboard and/or lay out a few on the flannel board with yarn. Coloring the enclosed region with chalk or colored material will emphasize what part must be considered to determine the area of a region.

IS THE CURVE CLOSED?

DOES IT ENCLOSE A REGION?

WHERE IS THIS REGION?

DOES THIS REGION HAVE AREA?

Hold up a piece of paper and ask if it has a boundary (Yes, its edges).

IS THERE A CLOSED CURVE HERE? (Yes.)

DOES IT ENCLOSE A REGION? (Yes.)

DOES THE REGION HAVE AREA? (Yes.)

Help the children find various surface regions as they work around the room: table tops, resting mats, book covers, etc. You might also ask them to trace these surface regions with their fingers.

Lesson 10: COMPARING AREA BY SUPERPOSITION TEST

In this lesson, the children see the need for comparing the areas of regions and design a scheme for doing so.

MATERIALS

- shapes cut from construction paper

or

- construction paper cut in 3 or 4 rectangles of varied sizes (enough for 2 pieces of different colors for each child) These sizes should range from approximately 4" x 6" to 9" x 1^"

PROCEDURE

For this lesson, each child will need two pieces of construction paper of different shapes. You can provide these in either of two ways: (1) You can cut the shapes yourself, making a variety of them (e.g., squares, triangles, circles, free forms) in different sizes and colors, then give each child two of them. Or (2) you may wish to give the children each two rectangles and a pair of scissors. Ask them to cut one shape from each piece, using most of their paper in doing so. This second method allows the children freedom in designing their own objects.

After each child has two objects, ask questions about the properties of the two objects. Some of these questions can be used mainly to stimulate thinking.

HOW ARE YOUR SHAPES ALIKE?

HOW ARE YOUR SHAPES DIFFERENT?

HOW DO THE AREAS OF THESE REGIONS COMPARE?

HOW CAN YOU TELL WHICH ONE HAS THE GREATER AREA?

Ask several children to describe to the class one of their objects in terms of its properties. Now ask a child to compare his object to another child's object which is similar in shape but grossly different in area.

Next choose two objects of similar shapes and almost equivalent areas.

DO THE REGIONS OF BOTH THESE OBJECTS HAVE THE SAME AREA?

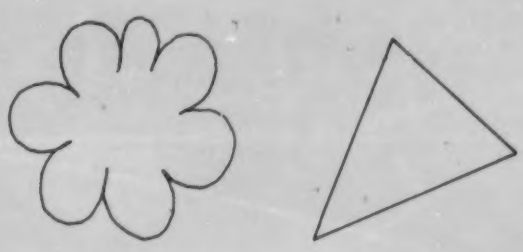
WHICH HAS MORE ROOM OR AREA?

HOW CAN WE BE SURE?

IS THERE A WAY SOMEONE CAN SHOW US?

If some of the suggested tests seem to compare length rather than area, remind the children that area involves the region within the boundary of the closed curve. If a child does not propose it, suggest yourself that the children put one shape on top of the other, hold them both together, and then observe the front and back of the combination. They will discover that when the region of lesser area is in front, some of the region with greater area can be seen around it, but that when the region with greater area is in front, the smaller area cannot be seen at all. Emphasize the fact that the region with greater area has enough room for the smaller to fit on it with room still left over. (For this demonstration be sure to choose shapes such that one covers the other completely.)

Repeat the questions on page 41 for other pairs of objects which are similar in shape and area. Then choose two objects of very different shapes which do not differ greatly in area.



DO THESE REGIONS HAVE AREA?

WHICH HAS MORE ROOM OR AREA?

HOW COULD YOU FIND OUT?

Try several pairs. The children should see that, in some cases involving dissimilar objects, they cannot use the superposition method of comparing for area because neither object will fit entirely upon the other. This problem will be handled in Lesson 14, but now just let the children think about the situation.

Lesson 11: ORDERING REGIONS OF SIMILAR SHAPE BY AREA

The purpose of this activity is to use binary comparisons in ordering more than two objects by area. Ordering is preceded here, as it was in Section 1, by an activity involving classifying a set of objects. Classifying requires a binary comparison, thus making more obvious the binary nature of the comparisons involved in ordering.

MATERIALS

- flannel board
- 4 rectangular flannel objects differing in area and color

--for each child --

- 12" x 18" drawing paper
- paste
- scissors
- Worksheets 1 and 2

PROCEDURE

[A] Distribute Worksheets 1 and 2. Have the class color and cut out the figures.

HOW ARE THE FISH ALIKE?

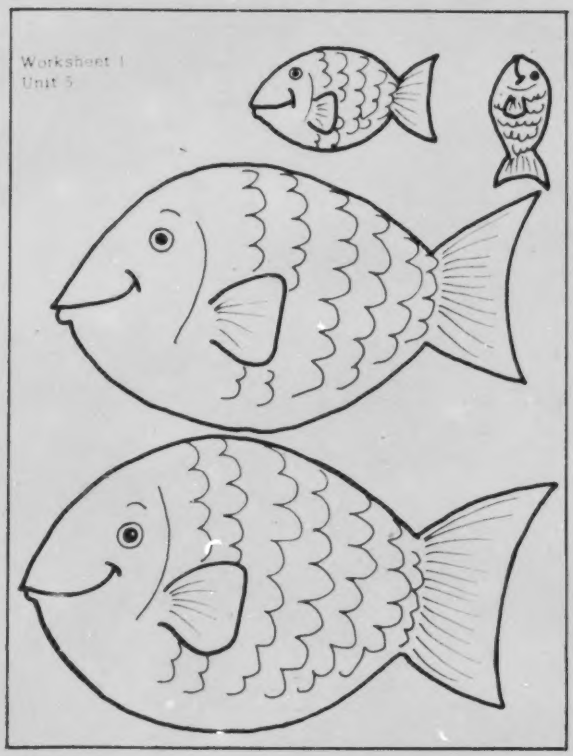
HOW ARE THEY DIFFERENT? (In area.)

WHICH FISH HAS THE GREATER AREA?

HOW CAN WE FIND OUT?

TRY TO FIND A WAY.

Have the children use the fish marked with "R" on Worksheet 2 as their reference fish to make two subsets, placing all fish with greater area than the reference fish on one side of their desks and all with lesser area on the other side.


Worksheet 1
Unit 5

Worksheet 2
Unit 5
R

As you observe their work, question individuals about their methods of determining the subsets. They should be comparing each fish with the reference fish, using a superposition test. Then ask:

WHICH FISH HAS THE GREATEST AREA OF ALL YOUR FISH?

HOW COULD YOU BE SURE?

WHICH FISH HAS A REGION WITH THE LEAST AREA OF ALL?

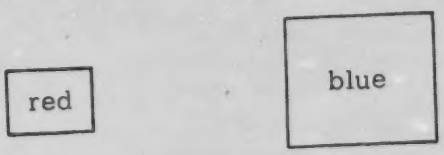
Emphasize, with your questions, the idea that each fish must be compared with each of the other fish either directly or indirectly (by means of another fish having an in-between area) using the idea of transitivity.

Distribute drawing paper. Tell the children they can imagine this is an aquarium for their fish and color the paper appropriately. Then have them decide in which direction they want to order the fish and paste the fish on the "aquarium" in that order. Notice whether the children are performing binary comparisons to do the ordering. Are they comparing the area of one fish directly or indirectly with the area of each of the others?

B (Optional) Choose any three of the four rectangular flannel pieces. Hold up two of them.

WHICH HAS A SURFACE REGION WITH GREATER AREA? (The blue.)

Put them on the flannel board, saying that you will put the one with greater area on the right or e.g., "nearer the window".

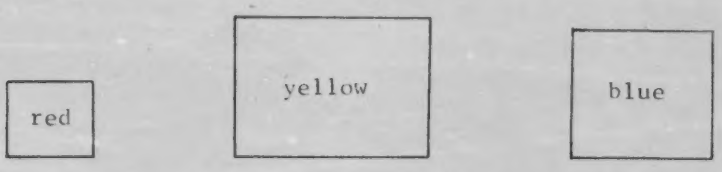


Pick up the third object and have a child compare it with one of the first two, e.g., the red.

IS THE AREA OF THE YELLOW REGION GREATER OR LESS THAN THE AREA OF THE RED REGION?

WHERE SHOULD WE PUT IT?

Have it placed in the proper location with respect to the red figure (but not necessarily with respect to the blue).



IS IT IN THE RIGHT PLACE?

DO WE NEED TO DO ANYTHING ELSE?

WHAT?

IS THE AREA OF THE YELLOW REGION GREATER OR LESS THAN THE AREA OF THE BLUE REGION?

You may wish to repeat this portion using yellow and either red or blue as the first two objects.

Keeping the three objects in place, bring out the fourth object, e.g., green.

HOW COULD WE FIND THIS OBJECT'S PLACE IN THE ORDER?

WHAT WILL WE HAVE TO DO TO FIND OUT? (Compare it with each of the other objects.)

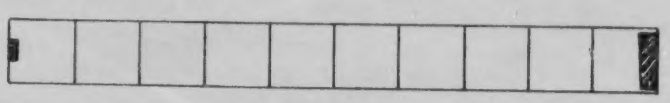
Lesson 12: AREA-ORDERING GAMES

Because of the quantity of materials required for large numbers to play at once, these games are probably best played by a small group of children in free time.

MATERIALS

-- for each group playing --

- playing-board made of a tagboard strip about 4-5 inches wide and as long as you wish. Mark each playing-board off in squares. At one end of the playing-board color a narrow strip with a magic marker. At the other end, make one with obviously greater area.



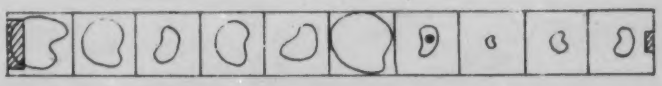
- 1 deck of shapes like those on p. 49, cut from heavy paper (8 sheets of these shapes are provided with the printed materials for this unit.)

PROCEDURE

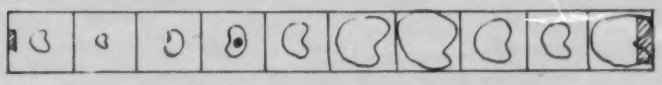
This can be played as a classifying or ordering game.

[A] Classifying: The first child chooses a reference object and places it in one of the middle squares. This shape should be marked in some way to indicate it as the reference object. A counter or paper token on top of it would be fine. Successive players draw shapes from the deck and place them in any space on the proper side of the reference object -- the side with the larger mark indicating greater area than the reference object and the side with smaller mark indicating lesser area. Play continues until all spaces have been filled or all cards are gone. If there are more cards than spaces on the board the player may remove one and replace it with his.

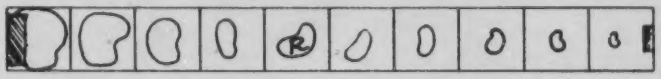
When play is finished the board might look like this:



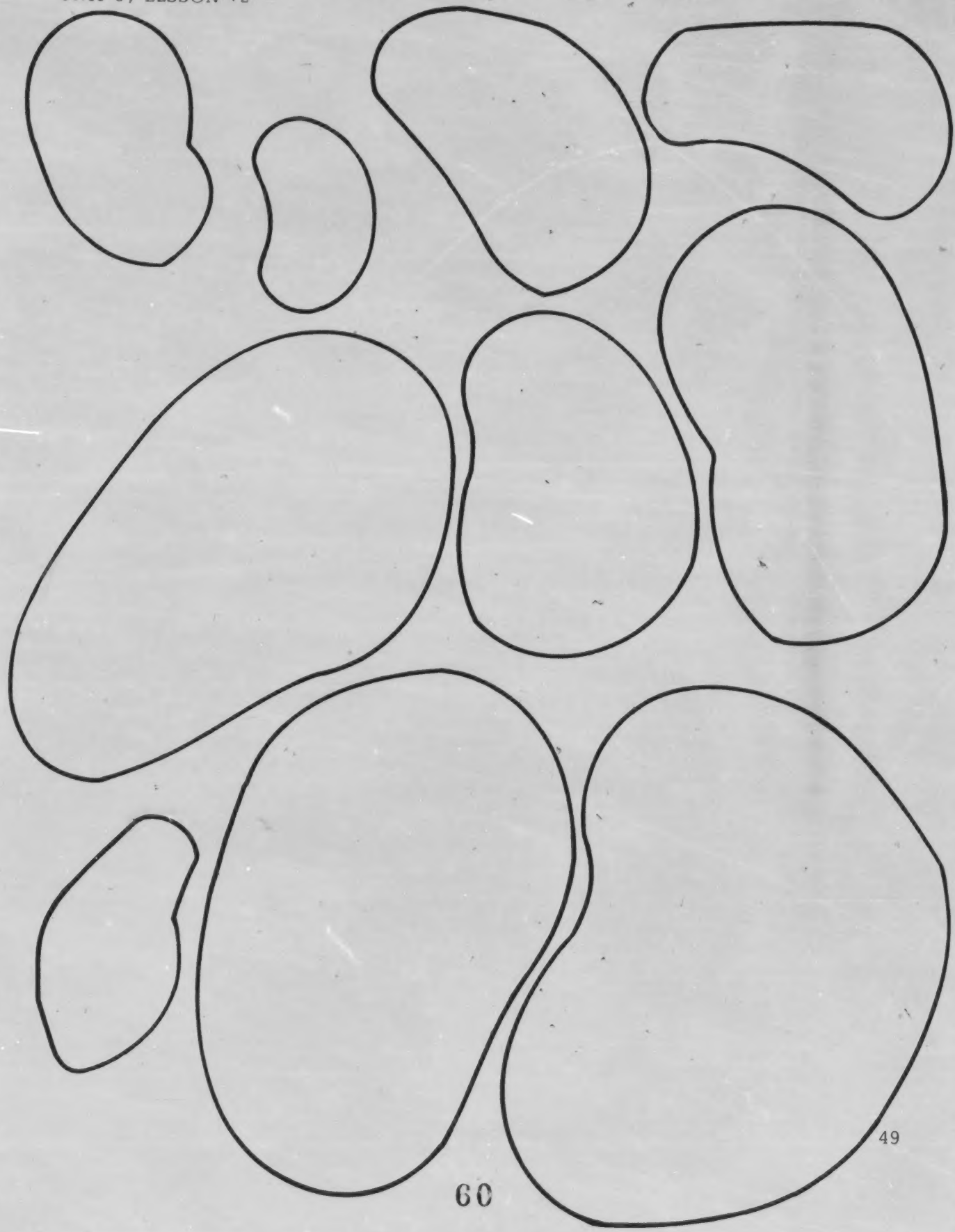
or this:



B Ordering: The first child again draws a reference card, places it in some middle space and puts a token marked "R" on it to show it is the reference. Successive players draw cards and place them on the proper side of the reference, this time establishing an order in each direction -- from the least area. Shapes already played will probably have to be moved from one space to another in order to place succeeding shapes correctly. When play is finished the board might look like this:



SHAPES TO CUT OUT FOR AREA-ORDERING GAMES



Lesson 13: INVARIANCE AND VARIATION OF APEA

Here the children see that where the areas of two regions do not change, the results of the comparison of their areas to each other will be the same, regardless of when or where the comparisons are repeated. They will also be shown an example of an area that does change (crepe paper before and after stretching). The variance is demonstrated by comparison with an invariant area before and after the stretching.

MATERIALS

-- for each child --

- 2 sheets of plain white paper of the same shape and area, for Activity A
- 1 red and 1 blue crayon
- scissors
- 1 piece of crepé paper and 1 piece of plain paper of the same shape and area, and 1 piece of crepe paper smaller than these, for Activity B

PROCEDURE

[A] Distribute the two sheets of plain white paper (of the same shape and area) to each of the children. Have them compare the regions of the two sheets for area. Have one child trace the boundary of the region on the chalkboard to "remember" what area those sheets had.

Then ask the children to move to a different place in the room with their papers.

WHICH SHEET HAS THE GREATER AREA NOW? (They still have the same area.)

COULD SOMEONE MOVE TO A PLACE WHERE THE COMPARISON WOULD NOT BE THE SAME? (No matter where we move in this room, we still find that the two sheets have the same area.)

Now have the children cut one of their sheets into two pieces and color one of the pieces blue and the other red.

WHEN PUT BACK TOGETHER WILL THE AREA OF THE COLORED PIECES BE GREATER THAN, LESS THAN, OR THE SAME AS THE OTHER (UNCUT) SHEET?

HOW DO YOU KNOW?

WHAT COULD YOU DO TO FIND OUT? (Try it.)

Have the comparison carried out and then have the children put their red pieces aside.

WHEN YOU COMPARE THE AREA OF YOUR BLUE SHEET WITH YOUR WHITE SHEET, WHICH WILL BE GREATER?

SIT DOWN ON THE FLOOR AND HOLD THEM. WHICH REGION HAS GREATER AREA NOW?

Next have all children put the blue piece aside and compare the red piece with the white sheet. Then ask them to fold up the white sheet.

NOW, IF YOU UNFOLD YOUR WHITE SHEET, WILL IT STILL HAVE GREATER AREA THAN YOUR RED PIECE?

TRY WAYS OF TESTING THIS.

Ask several children to show their methods. They should see that folding one sheet does not change the relative area of its surface region. The simplest way of showing this is to smooth out the folded sheet and compare the two again.

[B] Give each child one piece of crepe paper and one piece of white paper of the same area.

WHEN WE COMPARE THESE PIECES, WHICH HAS THE GREATER AREA? (They have the same area.)

STRETCH YOUR CREPE PAPER. NOW WHICH HAS THE GREATER AREA?

CAN YOU EVER MAKE THE TWO REGIONS HAVE THE SAME AREA AGAIN?

Give the children another piece of crepe paper which is smaller in area than the white paper.

WHICH PIECE HAS THE GREATER AREA?

CAN YOU MAKE YOUR CREPE PAPER HAVE A GREATER AREA?

Lesson 14: COMPARING AREAS OF DISSIMILAR SHAPE

The purpose of this lesson is to have the children discover and use a test for comparing the areas of dissimilarly shaped regions. Actually, this test is only a modified form of the superposition test in Lesson 10, but it does require the idea of the invariance of area under cutting, introduced in Lesson 13. In order not to make the test too difficult, do not use any regions that are very close in area size (except for reference regions).

MATERIALS

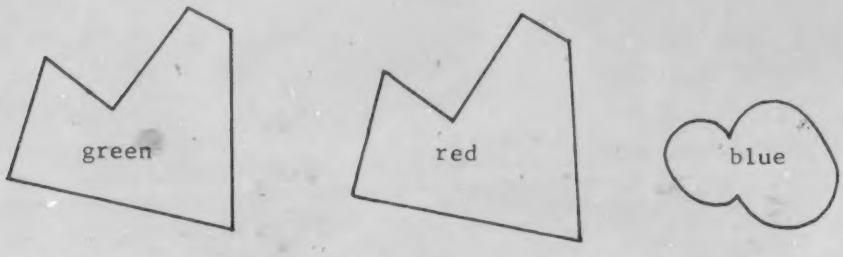
- 3 irregularly shaped objects of paper or flannel, 2 of the same shape and area and 1 of a different shape and less area (See diagram below.)
- 1 paper or flannel circle, 6" diameter
- 1 paper or flannel rectangle, 2" by 22"
- flannel board

-- for each child --

- Worksheet 3 (50 are provided)
- scissors
- crayons

PROCEDURE

[A] Show the children the three odd-shaped objects of colored paper or flannel.



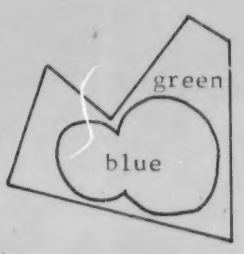
Have the objects described and similarities and differences noted.

CAN YOU TELL ANYTHING ABOUT THE AREA OF THE THREE REGIONS?

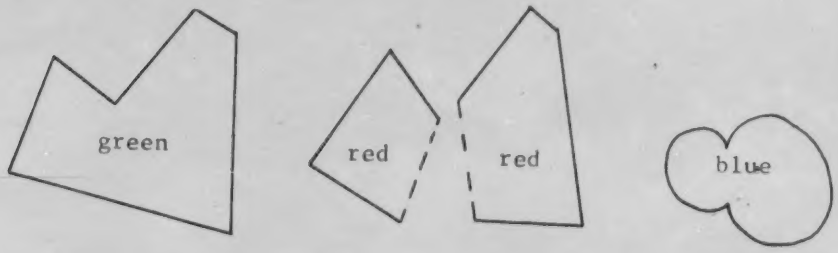
WHICH HAS GREATER AREA -- THE RED REGION OR THE GREEN REGION? (They seem to have the same area. We can put one on top of the other with nothing left over.)

THE GREEN REGION OR THE BLUE REGION? (The green.)

HOW CAN YOU SHOW THAT? (By placing the blue on top of the green.)



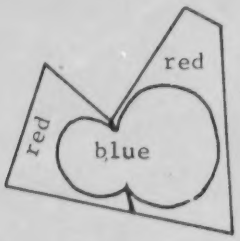
Then cut one of the larger objects into two pieces. For instance:



WHICH HAS GREATER AREA -- THE TWO RED PIECES TOGETHER -- OR THE GREEN PIECE? (They seem to have the same area.)

WHAT CAN YOU DO TO FIND OUT? (Put the red pieces together on top of the green one.)

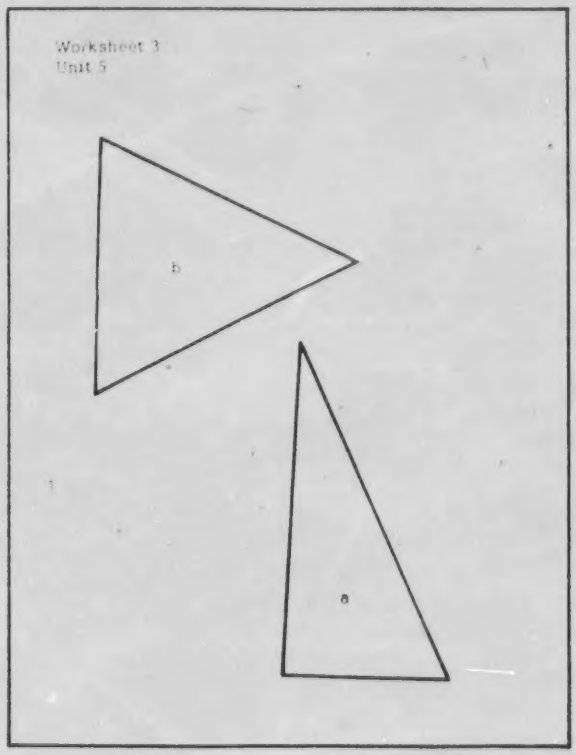
CAN YOU FIND OUT WHICH HAS THE GREATER AREA -- THE WHOLE RED REGION OR THE BLUE REGION? HOW? (By putting the red pieces together and placing the blue region on top of them.)



IS THE RESULT OF THE COMPARISON ANY DIFFERENT BECAUSE WE CUT THE RED REGION INTO PIECES?

DOES THE RED REGION STILL HAVE GREATER AREA THAN THE BLUE REGION?

B Distribute Worksheet 3. Instruct the children to color the inside region of each triangle a different color and cut out the triangles.



WHICH OF YOUR REGIONS HAS THE GREATER AREA?

CAN YOU GUESS?

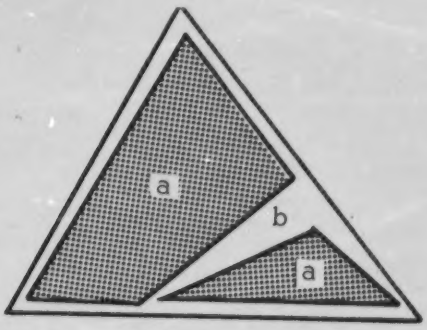
WHAT COULD YOU DO TO FIND OUT?

Let them try various methods, but at some point ask:

IS THERE ANY WAY YOU CAN LAY ONE REGION COMPLETELY ON TOP OF THE OTHER AND HAVE ROOM LEFT OVER? (No, since each object protrudes beyond the other.)

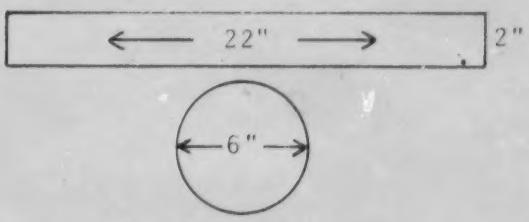
DO YOU THINK THERE IS SOME WAY WE COULD MAKE ONE REGION FIT ON THE OTHER?

Let the children try their own methods. One useful method would involve cutting one of the triangles (a) and placing the pieces on the other triangle (b).



If region a fits entirely within region b, then the area of a is less than the area of b. If some children cut the larger region first, encourage them to arrange and rearrange the pieces to see whether they can fit one region entirely on the other. If a child's pieces are such that he can reach no conclusive results, provide an extra worksheet so he can try another method.

C Show the large circular and rectangular regions.



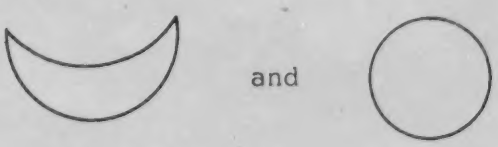
WHICH HAS GREATER AREA?

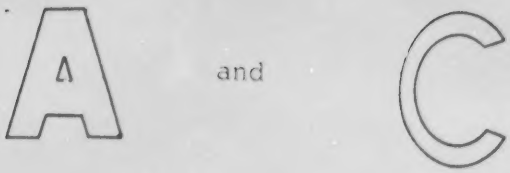
TRY TO ESTIMATE.

WHAT CAN WE DO TO BE SURE?

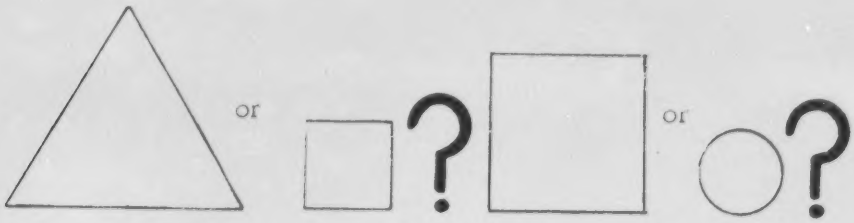
Carry out the tests which the children suggest. The best method is probably cutting off pieces of the rectangular region to cover the circular region. Also show that the circular region does not completely cover the strips.

D (Optional) The children might enjoy comparing the areas of various other dissimilar regions, such as those shown here. Cut the shapes from stiff paper.





E Show the children pairs of property blocks, pointing to one face on each and asking how they could find out which face has the greater area.



They should compare the two faces with a superposition test, that is, laying the blocks face to face and noting which face has room left over. Make sure that the faces chosen can be compared with a simple superposition, not requiring cutting.

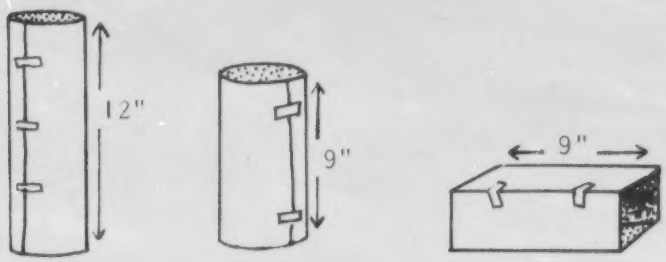
You might vary this by giving each child a block and having pairs of children compare their blocks for area. Here again you must be careful that the comparisons can be made by a simple superposition test. If a question comes up about how to make area comparisons when superposition does not work, let finding a method be a challenge to the children. One method for making such an indirect comparison is developed in Lesson 15.

Lesson 15: AREAS OF THREE-DIMENSIONAL OBJECTS (OPTIONAL)

The purpose of this lesson is to show the children that three-dimensional objects have surfaces whose area can be compared. It is not expected that the results of these comparisons will be very precise, but the children should understand how such a comparison is made.

MATERIALS

- Make 3 objects from 3 sheets of 9" x 12" paper of different colors (e.g: 1 red, 1 white, 1 blue), as pictured below.



RED WHITE BLUE

- 2 fairly large blocks of different shapes and area

PROCEDURE

[A] Show the children the red object.

DOES THIS OBJECT HAVE AREA?

Now show the white object.

WHAT REGIONS OF THESE TWO OBJECTS SHOULD WE CONSIDER? (All the surfaces.)

HOW COULD YOU COMPARE THE AREA OF THE RED OBJECT WITH THE AREA OF THE WHITE OBJECT?

There are a number of possibilities but perhaps the easiest is to remove the tape and lay the objects out flat. Repeat this strategy for the comparisons of the white object with the blue, and for the red with the blue.

B Show the children the two blocks of different shape and area. Ask if each of the blocks has an area and which parts they must consider in finding the area of all its regions. Ask questions similar to the following until they see that a useful method might involve tracing each face of each block on paper and cutting up one drawing to superpose on the other.

HOW COULD WE FIND OUT WHICH REGION HAS THE GREATER AREA?

COULD WE USE THE SAME METHOD WE USED FOR THE OBJECTS MADE OF SHEETS OF PAPER? (No.)

WHAT COULD WE DO TO SHOW HOW MUCH AREA EACH BLOCK HAS?

WHAT PARTS MUST WE CONSIDER?

COULD PAPER HELP US?

Have several children help you demonstrate the method for the class. Since this is a fairly complex concept, all children should not be expected to grasp it.

SECTION 3 COMPARING VOLUME

PURPOSE

This section contains activities designed to develop:

- An intuitive notion of volume as a measure of a spatial region.
- Techniques for comparing volumes. The child should be able to choose and use an appropriate test for comparing the volume of two objects.
- An understanding that if the volumes of objects are invariant, then the results of comparing them should remain constant. As an example, the child should see that the pouring of liquids from containers of one shape into ones of a different shape does not alter the relative volumes of the liquids.
- The skill of ordering two or more objects by the property of volume. A child should be able to make binary comparisons of volume using an appropriate technique to establish the order.

COMMENTARY

In this unit, volume is presented as the measure of the interior of an object -- the measure of a spatial region enclosed by a simple closed surface. One should remember that all objects occupy space; the one that occupies more space has the greater volume. The shape of two objects can be different, and yet the spatial regions enclosed by their surfaces can have the same volume. The weight of an object is not necessarily an indication of its volume. For example, a five-pound piece of granite occupies less space -- has less volume -- than a five-pound bag of popcorn.

The property of volume will be compared in four ways:

- By observation one can often tell which of two objects has the greater volume.
- By fitting one object completely within another object, one can see that the former object has less volume than the latter.

- By filling one object and then pouring the filling material into another object one can determine which of the two objects holds more.

- By comparing the amounts of liquid displaced by two objects one can determine which has greater volume. (This method will not be identified explicitly as a way of comparing volume.)

In Unit 12, Measuring with Reference Units, a quantitative procedure for measuring and ordering the volume of objects is explored.

Lesson 16: INTRODUCING VOLUME

This lesson is designed to develop an intuitive notion of volume as a measure of a space region. The activities also focus attention on a need to compare the volume of objects. The children handle and examine materials with very little specific direction. After this exploration, there is a group investigation and discussion.

A story, "Pete's Popcorn Problem," is outlined in this lesson. It is intended to provide motivation for investigating the volume property of objects. You may want to substitute any story which provides the same stimulus for investigation. If you do this you may need to substitute, for the popcorn, another filling material to suit the different situation.

MATERIALS

- 2 containers of different volumes with deceptive shapes (The children should not be able to tell at a glance which container would hold more.)
- popcorn (or similar filling material) in 1 large container

-- for each group of 3 or 4 children --

- 1 tray with 2 small containers of different volumes and shapes (e.g. 6 fl. oz. juice can, 3 oz. Jell-O box) (Ask the children to bring these from home.)
- 1 plastic container (12 oz.)

PROCEDURE

With the two large containers exhibited for the children, show them the popcorn and ask whether they like it. Tell them that you know a story about another child who also liked popcorn very much. Then proceed with the story, "Pete's Popcorn Problem," or any similar one you have selected.

PETE'S POPCORN PROBLEM

Pete liked popcorn very much! He ate popcorn plain and buttered. He liked popcorn balls and caramel popcorn. He ate popcorn while he watched TV and as a snack just before he went to bed. He liked going to the movies because he could buy an especially big box of popcorn there! Pete would even eat popcorn for breakfast -- if only his mother would let him.

One day Pete's father told him he had a special surprise for him.

"What is it, Daddy? What is it?" Pete asked.

"You will find out on Saturday. I know you will like this surprise very much!"

Pete woke up early on Saturday. So did Daddy.

"Today's Saturday, Dad. Tell me what my surprise is -- please!" Pete asked.

"We will drive to the surprise. Come on! Let's get started!" Father answered.

Soon they came to a big, big building. Pete saw a sign. He didn't know many of the words on the sign but he did know one! POPCORN!

"A friend of mine owns this popcorn factory," Daddy said. I told him how much you like popcorn. He said we could see workers here getting all kinds of popcorn ready for people to buy."

Pete and his father met the man who owned the factory and he showed them the popping machines. Then Pete saw how they packaged the popcorn and how they made popcorn balls.

When they had seen all these things the man said, "Would you like to take some popcorn home?"

"Oh yes!" Pete answered.

"You must decide one thing. Here are some different containers." (Point out the two containers you have set out.) "You may fill one of them with popcorn."

Pete looked at the containers. They were different shapes. He wanted to be sure he took as much popcorn as he could. He knew he had a problem!

WHAT DID PETE WANT TO FIND OUT? (Which container would hold the most popcorn.)

HOW COULD HE FIND OUT? Ask the children for suggestions.

Tell the children that they should all be thinking about how they could help Pete solve his problem. Tell them that you will be giving them some objects that may help them.

Now arrange the children in groups and give each group the set of two small unfilled containers and the plastic container filled with popcorn.

Ask the children to find out which container (juice can or Jell-O box) they would use if they were going to take home some popcorn. As they are doing this, watch each group, noticing the various methods they use. You will want the children to get the idea that the various containers hold different amounts. Some children may even see that if they fill one container and then pour that amount into the second container, they can tell which one holds more and how much more.

The questions you ask might be similar to those listed on the next page (depending upon the situation).

1. HOW DO YOU DECIDE WHEN TO STOP FILLING A CONTAINER? (They should imagine a flat top on each container.)

2. WHICH HOLDS MORE POPCORN -- THE JELL-O BOX OR THE JUICE CAN?

3. If the children have filled both containers with some popcorn ask, CAN YOU FIND OUT ANYTHING ABOUT THE CONTAINERS USING ONLY THE POPCORN NEEDED TO FILL ONE CONTAINER?

4. POUR THE POPCORN FROM YOUR JUICE CAN INTO THE JELL-O BOX. WHAT DO YOU NOTICE?

Collect the materials when you feel the children have had sufficient time for their investigations and bring the class together again. Show the large containers and ask for some ideas about how to help Pete take home the most popcorn. The children's first thought may be just to compare the containers by observing. If it is, ask them to suggest a method of testing. Try several suggestions. The most efficient is probably to fill the container which they think holds more and then pour the contents of that container into the other one. If the first does hold more, some of its original contents will spill out or be left over. If it does not hold as much, the second container will not be completely filled.

The first time the children try filling a container, ask how they know when to stop filling. Guide them to realize that they must agree on where the enclosing surface is to be located, if it is not concretely represented, as in this case. By agreement, have the children see that it is convenient to imagine a top or cover for each container.

WHAT DID YOU FIND OUT? (We found out which container holds the most popcorn.)

WHEN WE TALK ABOUT HOW MUCH SOMETHING HOLDS WE ARE TALKING ABOUT <u>VOLUME</u>.

WHICH CONTAINER HAS THE GREATER VOLUME?

Lesson 17: ORDERING CUBES BY VOLUME

This lesson provides experience in ranking two or more objects according to volume. The children will also learn about another test for volume.

MATERIALS

- Minnecubes

PROCEDURE

[A] Show the children the 2 1/2" and 3" Minnecubes. Hold them far apart so that the edges cannot be compared directly.

WHICH HAS GREATER VOLUME?

HOW COULD YOU SHOW THIS WITHOUT FILLING EITHER MINNECUBE?

If they do not suggest putting one inside the other, just open up the 3" cube and place the 2 1/2" one inside, close the flap and show the children all sides of the cube.

WHICH HAS GREATER VOLUME?

Briefly discuss with the children the idea that if one object can fit entirely within another object, the enclosing object has the greater volume.

Then compare other pairs of Minnecubes (1/2" - 1", 1 1/2" - 2", 2 1/2" - 3", 3 1/2" - 4",) leaving them inside each other in view of the children.

Now show them three Minnecubes (3", 3 1/2", 4").

COULD SOMEONE PLACE THESE CUBES IN ORDER BY VOLUME?

As they determine this ordering, emphasize that they must compare each cube with both the others (i.e., they must show that the 3" cube has lesser volume than the 4" cube and the 4 1/2" cube).

Have groups of children order entire sets of the Minnecubes, placing them in a "train" arrangement such as is used in Activity B.

B Game: Charlie Choo-Choo

Arrange groups of three or four children along one side of the classroom. Place one set of Minnecubes in a mixed arrangement across from each group about ten to twelve feet away or on the opposite side of the room. Take the 2 1/2" cube from each set and place one of these by each group of children. Tell the children that they will be building a train called "Charlie Choo-Choo." You have given them the middle car. Their task is to arrange the rest of the cubes (cars) in order by volume to complete the train. The children in each group will take turns in looking at the train they are building, deciding which car (cube) is needed (on either end) and getting the proper cube from the mixed set. As the train is built, a cube which is not in the correct place should become obvious because of the irregularity in the line of blocks.

A variation of this game would be to build the train out of sight of the set of blocks such as behind the teacher's desk or in the coat hall.

The children might also enjoy playing with commercially produced nesting blocks or "Kitty in the Keg" (Child Guidance) sets.

Lesson 18: VOLUMES OF SOLID OBJECTS

The children have now experimented with two methods for comparing volume -- filling and enclosing. Both of these methods treat volume as a property of an object measured by filling up the inside. Neither of these methods is satisfactory for comparing the volume of solid objects.

One satisfactory test involves comparing the space one solid object takes in water with that taken by another object. It does not seem desirable to introduce such a test explicitly here because this method and the other two do not seem obviously related. For instance, filling one box and pouring that material into another empty box does not seem related to putting two stones, one at a time, in water and comparing the two new water levels. They could be more closely related by filling two empty boxes and comparing the amount of space each box then took in the water, but this seems too indirect for this age level.

It is desirable for the children to see that objects do take up space in water, although this is not introduced as a way of measuring volume, as such. They will also see that changing the object in some ways does not alter the amount of space the object displaces in the water.

MATERIALS

- pitcher
- crayons
- plasticine of at least 3 different colors

-- for each 2 children --

- 1 tray
- 1 plastic container (tall, 12 oz.)
- 2 pieces of plasticine of different colors, shapes and volumes

PROCEDURE

[A] Give each team of two children a tray with a plastic container partly filled with water, and the two pieces of clay described in the materials list. (To reduce the amount of splattered water, some teachers insert a stick into each lump of clay; others insert a paper clip tied to a string.) Tell the children to put the objects in the water one at a time. As they do this, ask questions similar to the following:

WHAT HAPPENS WHEN YOU PUT ONE OF YOUR PIECES OF CLAY IN THE WATER?

WHICH PIECE OF CLAY MAKES YOUR WATER RISE HIGHER?

When you feel the children have had sufficient time for investigation, collect the materials.

[B] Show the children two unequal amounts of clay of two different colors. Roll one into a ball and the other into a long thin cylinder. Ask the children to predict what would happen if the objects were put in the water in a pitcher or a glass. Through questioning, predicting, and testing they should observe that:

- The rising water shows that the object takes space in the water.
- Different sized objects cause the water to rise to different levels.
- There must be some way of marking or recording the level for each object in order to compare the various levels.
- Changing the shape of an object makes no difference in its volume.

It is best in a situation like this to guide the children in suggesting the operations to be performed, always keeping the desired concepts in mind. The activity might proceed in this way:

Show the clay ball and clay cylinder.

WHAT WOULD HAPPEN IF THESE WERE PUT IN WATER?

Children suggest putting both in water.

WHAT HAPPENS? (The water rises.)

Take both objects out of the water.

IF JUST THE BALL IS IN THE WATER, DOES THE WATER RISE TO THE SAME LEVEL AS WHEN BOTH OBJECTS ARE IN THE WATER? (No.)

WOULD THE CYLINDER MAKE THE WATER RISE TO THE SAME PLACE AS THE WATER WOULD RISE TO IF THE BALL WERE PUT IN?

HOW CAN YOU FIND OUT?

Take out the ball and put in the cylinder.

NOW CAN YOU TELL WHICH MADE THE WATER RISE HIGHER? (We can't be sure.)

HOW COULD YOU REMEMBER HOW HIGH THE BALL MADE THE WATER RISE? (We could mark the water level.)

Take the cylinder, put in the ball, and mark the water level with a felt-tip pen or rubber band.

NOW HOW CAN YOU FIND OUT IF BOTH MAKE THE WATER RISE TO THE SAME PLACE? (By taking the ball out and putting the cylinder into the water.)

Take out the ball, leaving the mark, put in the cylinder and compare the levels. Then reverse the shapes, making the ball into a cylinder and the cylinder into a ball. Refer to the objects from now on by their color, since this property has remained the same while the shape has changed.

WILL THE RED CLAY -- or whichever color is appropriate -- STILL MAKE THE WATER GO HIGHER? WILL IT TAKE UP MORE SPACE IN THE WATER? (Yes.)

HOW CAN WE FIND OUT?

C Show the children three balls of clay. The balls should be of different colors and of substantially different volumes. Ask a child to put one ball in the water. Mark the level. Have other children put in the same ball and see that the water rises to the same level each time. Repeat this procedure for the other two balls. Label each level with a mark the same color as the clay.

Wipe off the clay after it is removed from the pitcher so that water will not be kneaded into the clay. If you haven't already done so, it may be desirable to bury a paper clip with a string tied to it in each clay ball to help in pulling the ball from the water. The children may want to see that the paper clip and string do not cause the water level to rise a detectable amount.

Assign three different children to make a figure from each of the clay balls, using all of the clay. When they are finished, ask which properties of the clay have changed and which properties have not. After the children have mentioned several changes, ask if the property of the clay that makes the water rise to a particular mark has changed. Have the children test the figures in the water and compare the water level of each object to its original mark.

If too much water is lost from the container in removing the clay, new water may be added periodically to keep the water level (without clay) at its original level. The children should see why this is necessary.

D Arrange the children in three groups. Divide one of three balls among the children in each group. Tell each child to make an object from his piece. When the products are finished, discuss which properties have changed or not changed. The number of pieces from each ball should be noticed at this time. Ask whether the clay had the same number property before. Point out that the number of pieces was "one" before. If no one noticed the number property, it should help the children understand that it is difficult to name all the properties of an object.

Now ask which group's clay takes the most space in the water. Emphasize that you want to find out about the space all of the

pieces of each color take together. Ask how the children could find out the amount of space each color of clay takes. Have each group put their pieces in the water and observe the level. It is unlikely that the children will notice the relation between the space of the pieces together and that of the original ball. Commend anyone who notices that the previous marks on the pitcher are still useful.

Lesson 19: ORDERING OBJECTS BY VOLUME

This lesson gives the children more experience in ordering volume. They use two of the tests for volume that they have used previously: enclosing and filling.

MATERIALS

- sets of Minnecubes
- 3 jars or bottles of different shapes
- pitcher partly filled with water

PROCEDURE

[A] Show the children three Minnecubes (3 1/2", 4", 4 1/2") and ask whether they can arrange them in order by volume. Have a child test his visual estimate by nesting the Minnecubes (the enclosing test) to be sure he can place them in the right order. Have the cube with the least volume placed to the left side of the children, the largest to their right. This orientation is consistent with later work on the number line.

[B] The test for volume here is by filling. Show the children two bottles or jars which are very different in shape but not much different in volume.

WHICH CONTAINER HAS THE GREATER VOLUME?

HOW COULD WE COMPARE THEIR VOLUME?

After various methods have been tried, fill one of the containers with water. The water may be dyed with food coloring in order to make observation easier. By pouring the water from one container into the other, the children should be able to tell which has the greater volume.

Now ask how a third bottle would compare with the other two. Let the children perform the test and place the three bottles in order by volume -- placing the one of greatest volume on the right.

SECTION 4 TIME DURATION AND TIME ORDER

PURPOSE

- To introduce time duration and time order of events.
- To give practice in the correct use of such terms as "longer," "shorter," "greater than," "less than," "before" and "after," when referring to time.
- To make tests determining which of two events has the greater time duration if the events start together, or end together, or if one event begins before the other event and ends after it.

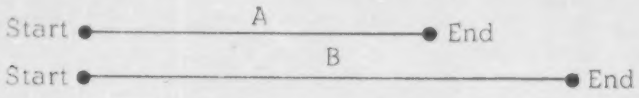
COMMENTARY

In this unit the terms "greater than" or "less than" have already been used to compare the measures of a common property of two objects. The terms "greater than" and "less than" are quite general and may be applied to most comparisons. The property which is being compared is sometimes unclear without further description. However, special comparative words can be used which imply the property that is meant. For example, "longer" and "shorter" suggest the property of length, as distinguished from "larger" and "smaller" which might apply to other measures of size (e.g., area, volume). Yet even the terms "longer" and "shorter" might mean something takes a longer or shorter "time" just as something is longer or shorter "in space."

An event is here considered as a particular section of experience in which some change occurs. Thus typical events might include counting to ten, walking around the room, singing a song, drawing a picture, or blinking an eyelid. They may be of relatively long duration (the melting of an ice cube) or of brief duration (the swing of a pendulum). An event is related to time as an object is related to space, i.e., an object has spatial extension and an event has temporal extension.

Throughout this section diagrams will be used to illustrate certain points in the commentary. The diagrams will use

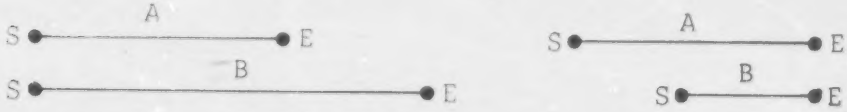
line segments which will represent durations of events. For example:



The above diagram indicates that event B had a longer duration than event A. Also the diagram makes it clear that both events started at the same time and that event A was finished before event B.

What property of an occurrence (activity, event) do we mean when we say it takes "longer" than some other one? This property of an event, which is analogous to that of length in space for an object, we can call its "duration" or "time duration." The duration of some events may be compared in the same way that lengths of objects are compared. The lengths of two objects may be directly compared by physically placing the two objects together side by side with one pair of ends aligned, and that object extending beyond the other in space has the greater length. Similarly, durations of concurrent events may be compared directly if both events begin or end simultaneously. That event which continues after the other has stopped, or starts before the other has started, has the greater duration.

Diagramatically the above situations can be shown:



Another analogy between duration and length is that of apparent sameness. If the observer cannot detect a difference in the duration of two events, the events will appear to have the same duration. But whether two events will or will not appear to have the same duration depends on the sensitivity of the measuring devices used for the comparison. For example,

horseracing fans in the grandstand may think two horses both finished at the same time, but the judges -- with the help of photographs -- may be able to observe a difference in the durations. The "same duration" is analogous to the "same length" -- it depends on the sensitivity of the length measuring device. The word "duration" is used instead of "time" in order to avoid confusion with another time-related idea.

Besides the duration of events, the order in which these events occur can also be considered. Placing events in order by considering when they occur is called "time ordering." Suppose one observes two events -- washing dishes and eating dinner. Two time-related observations can be made regarding these events: (1) that washing dishes has a lesser time duration and (2) that eating dinner occured before the other event. The two time-related properties of this section are time duration and time order.

The time ordering for property events which is based on before-after relationship, may be determined in terms of memory. If one event is perceived directly, and the other is remembered, the remembered event occurred before the directly perceived event. An observer notes three events, A, B, and C. A occurred before B, and B occurred before C. The before-after relationship is transitive and one can infer that A occurred before C. The sequence A, B, C is said to be time-ordered.

NOTE: Before starting the next Lesson, read Lesson 23 and begin planting a few seeds each day according to the instructions given there. If you do this now, seeds will have germinated and be ready when you need them.

Lesson 20: UP IN THE MORNING

Here you use a picture story, "Up in the Morning," to lead the children to consider how events are arranged in their time order. The children learn to use correctly such words as "before" and "after" to denote time relationships.

MATERIALS

- 8 panels of story, "Up in the Morning," each printed on heavy paper

PROCEDURE

The picture story shows the activities of a Minnesota family on a wintry morning. Mix up the pictures. Then show them -- one at a time -- and have the children help you place them in the chalk tray in the proper time order. The time order chosen by the children may not be the same as the arrangement of the pictures in this manual, because events in their homes may occur in a different order. Select any picture first and show it to the children. Do not read the verses to them at this point. Ask questions that will require the children to discuss what is taking place in that particular scene, e.g.:

WHAT IS THIS BOY DOING?

WHAT IS THIS GIRL DOING?

Have a second picture placed in order with respect to the first one -- to the left if the event pictured happened before the first, to the right if it happened after. Repeat the steps -- discussion, comparison and ordering -- until all eight pictures have been arranged. Emphasize the fact that to determine its place in the sequence, each new event must be compared to each of the previously ordered events.

DO WE EAT BREAKFAST <u>BEFORE</u> WE GET DRESSED?

DO WE BRUSH OUR TEETH <u>BEFORE</u> OR <u>AFTER</u> EATING BREAKFAST?

After all the pictures have been placed in a time order decided by the class, discuss the panels in sequence and read the verses to the children.

We stretch and yawn to chase sleep away,

Then it's UP IN THE MORNING to start the day.

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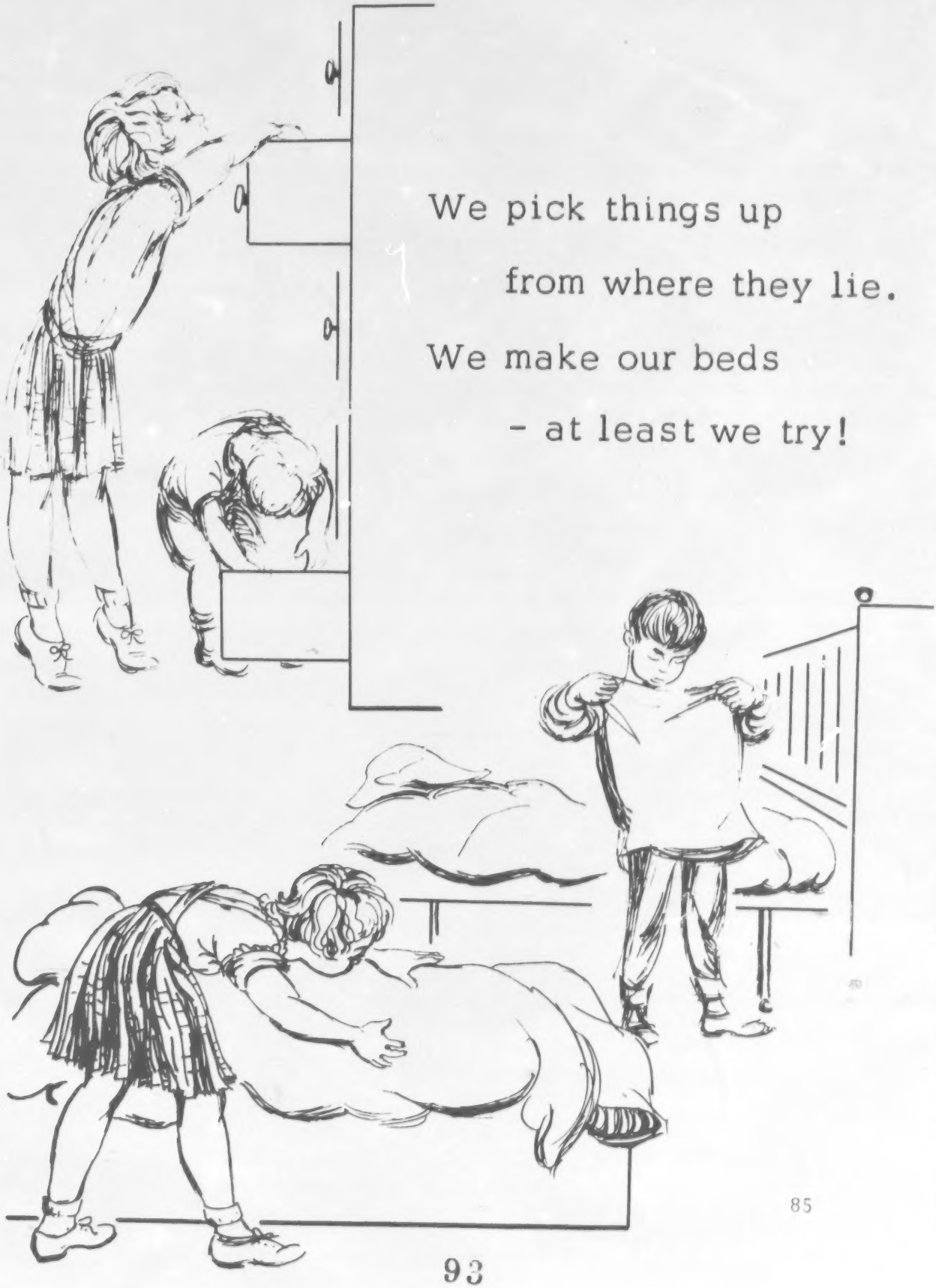
We put on our clothes - no time to lose,

Button our buttons and tie our shoes.



We pick things up
from where they lie.
We make our beds
- at least we try!

We smell the food that's on the table
We run to eat as soon as we're able.

We wash and brush till we're clean and neat.

Look us over from heads to feet.

Hurry now, it's almost eight.

Quick! Boots and mittens before we're late!

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We stop and think - What did we say

That we'd be sure to bring today?

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Are we ready?

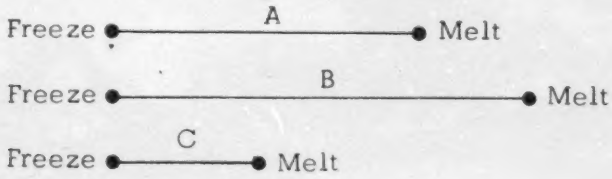
Is everything done?

Open the door

- it's time to run!

Lesson 21: COMPARING DURATIONS OF EVENTS

The purpose of this lesson is to help the children observe and describe events in terms of their durations and to lead the children to discover that durations of events may be compared. The case in this lesson would be:



PROCEDURE

Explain to the children how to play the game, "Statues." (Any bodily activity may be used in the game such as waving the arms and hands above the head, marching in position, simple rhythmic calisthenics, etc.) Have them start moving around and at your command, "Freeze," ask them to hold their positions as rigidly as statues. Then have them relax when you say, "Melt." This game provides an excellent opportunity to introduce the word "duration." Explain that the time during which each one remains a statue is called a "duration." Use the word frequently so that the children become used to it.

Have the youngsters "freeze" and maintain the position as long as they can. When a child moves, ask him to sit down. Conclude the game when only a few are left "frozen." The object of the game is to hold the statue position for as long a duration as possible.

After the game ask the children such questions as:

WHO FROZE AS A STATUE FIRST?

WHO FROZE LAST?

They should all have frozen at the same time. However, because they are all participating and not simply observing, they they may not see all the events of freezing as simultaneous. If this difficulty arises, ask:

DID ALL OF YOU FREEZE WHEN I SAID "FREEZE"?

You may agree with the children that some were slower than others to freeze, but if the differences were negligible, say so.

WHO SAT DOWN FIRST?

WHO WAS NOT A STATUE WHEN EVERYONE ELSE WAS A STATUE?

WHO WAS FROZEN AS A STATUE FOR THE LEAST DURATION?

Optional questions:

WHO SAT DOWN LAST?

WHO REMAINED FROZEN AS A STATUE WHEN NO ONE ELSE WAS A STATUE?

WHO WAS FROZEN AS A STATUE FOR THE GREATEST DURATION?

Lesson 22: "QUESTION TIME," A GAME COMPARING DURATIONS

In this lesson, the children compare durations of greatly differing magnitudes by playing "Question Time."

PROCEDURE

Ask the children questions about durations that are relatively easy to compare. Some children may also enjoy making up their own appropriate questions. Suggestions are:

YESTERDAY DID YOU SPEND MORE TIME AT SCHOOL OR AT HOME?

DID IT TAKE YOU LONGER TO PLAY STATUES OR TO REST?

HAS IT BEEN A LONGER TIME DURATION SINCE YOU WERE BORN OR SINCE I WAS BORN?

HAS IT BEEN A LONGER TIME DURATION SINCE YOU WERE BORN OR YOUR BABY SISTER?

WHO IS THE OLDEST CHILD IN YOUR FAMILY?

WHO IN YOUR FAMILY IS THE OLDER -- YOUR SISTER OR YOU?

Compare a five year old with a six year old.

WHO IS THE OLDER?

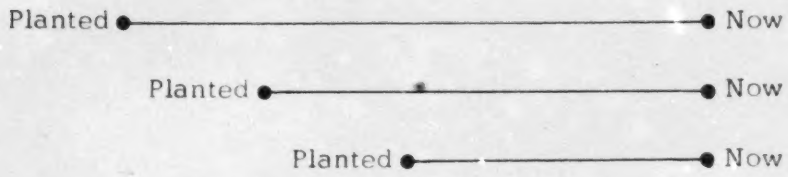
WHO HAS BEEN LIVING LONGER?

Of two children who both are now six years old, ask:

WAS THERE A TIME WHEN ONE WAS FIVE AND THE OTHER SIX? The children may experience difficulty in answering this problem because they do not know the order of the months. This is an opportunity to introduce the subject, or you may wish to leave the problem unsolved at this time.

Lesson 23: COMPARING GROWTH OVER LONG DURATIONS

In this lesson the children compare long durations of events which end together. They will also note several properties of objects which may change with time.



MATERIALS

- 2 kinds of seeds (about a dozen of each). Field corn and cowpeas have been used most successfully. If these are not available, use lima beans, pumpkin, squash or radish seeds.
- 6 to 8 clear plastic cups or glasses with foam packing or kitchen paper toweling (not the tan lavatory type which may be toxic to seeds) or 6 to 8 DisPo plastic seed pouches. DisPo pouches may be ordered from Scientific Products, 3846 Washington Avenue North, Minneapolis, Minnesota 55412; telephone: 529 - 7735, Area Code 612.

PROCEDURE

A This is the planting activity which should have been started just prior to teaching Lesson 20 so that the seeds will have germinated in time for use in Activity B. The instructions are:

1. First day: Plant 4 to 5 corn seeds in a pouch. Add 1 to 1½ inches of water to the pouch. Then tape it by its edges to a wall or to the window (if the winter weather is not too cold).

   Below these, plant 4 or 5 cowpea seeds.

2. Following days: Plant 4 or 5 seeds of each kind on each day, until all are planted. Place each day's pouches to the left of the preceding ones.

3. Let the children help plant and water the seeds. Encourage them to observe the plants daily, watch the seeds sprout, discover roots and shoots.

If you do not have seed pouches, use clear plastic containers or water tumblers for the plantings. Press each seed against the inside of the container with a wad of wet paper toweling. Place the seeds <u>against the sides</u> of the containers (not at the bottom), so that children can see the seeds as they germinate. If a single piece of paper toweling doesn't keep the seeds against the sides of the container, stuff another piece in the center of the jar. The drawing shows the arrangement. The paper toweling must be kept moist. If it dries out too quickly, place an index card over the top to slow down evaporation.

Seeds germinating in tumbler.

B Point to the seeds that have been growing for the longest duration.

DID WE PLANT THESE BEFORE OR AFTER THE OTHERS?

WHAT IS DIFFERENT ABOUT THEM?

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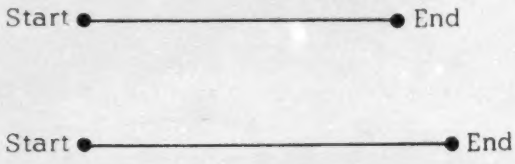
Note changes with time. Height changes of corn plants are most easily observed. Direct the children's attention to the root hairs which are clearly visible. Individual variation for seeds planted at the same time should also be noted by children. They may want to try to explain why some individual plants are taller than similar ones which have been growing for a longer duration.

Lesson 24: OBSERVING THE TIME ORDER OF EVENTS

This lesson deals with the comparison of events starting together and having short durations. The children will also observe time order of events which are in close succession.



MATERIALS

- 2 sheets of paper for you and 2 for each child. These sheets should be 6 to 9 inch squares, or if there is an interest in paper folding, 8½ x 11" yellow second sheets. (Most pulpy writing paper breaks too easily when folded.)
- (Optional) book Paper Folding for Beginners, by William D. Murray and Francis J. Rigney, Dover Publications, Inc., New York City, N. Y., 1960 ($1.00)
- a penny and a feather
- a pair of objects for each child; one that flutters to the ground, such as a leaf or maple seed, and one that falls quickly, such as a twig or pebble. (You might ask the children to provide these.)

PROCEDURE

It may not be necessary to do both Activity B and Activity D. Activity E is a preparation for quantitative measurement of duration which the children study later.

A Show two objects made of similar pieces of paper. One may be an origami paper-folded bird or kite, or just an unfolded sheet. The other should be something that falls more quickly, such as a paper glider. (Most classes will have at least one child who is expert in making gliders.)

IF I DROP THESE TWO OBJECTS, WHICH WILL TAKE A GREATER DURATION TO FALL TO THE FLOOR?

HOW CAN I FIND OUT? (By dropping them.)

Drop first one object, then the other -- releasing them about four feet above the floor.

CAN YOU TELL WHICH OBJECT TOOK LONGER TO FALL TO THE FLOOR?

Since you released one object before the other, the children should not be able to convince you that they could tell which object took longer to fall. Instead they should see and suggest that you drop both objects at the same instant. Carry out this suggestion. If both objects reach the floor so nearly together that discrimination between time durations is difficult, accept as a suitable answer, "Both objects had about the same time duration for falling to the floor."

B Give each child two pieces of paper.

CAN YOU CHANGE ONE OR BOTH OF YOUR PIECES OF PAPER SO THAT ONE TAKES A LONGER DURATION THAN THE OTHER TO FALL TO THE FLOOR?

Some children may need help in making birds, planes, gliders, etc. Show them how in small groups, and supply extra paper to children who do not like their first efforts. Some children may discover that leaving one sheet flat and crumpling the other into a ball produces an easily observable difference in duration time for falling -- if they release the flat sheet horizontally. If there is time, you may want to encourage the children to color their objects with crayons.

C Show a penny and a feather.

CAN SOMEONE TEST TO SEE WHICH TAKES LONGER TO DROP TO THE FLOOR?

D Give the children each two natural objects, such as a leaf and a small twig (or a maple seed and a pebble). Have them test to see which takes longer to fall to the floor.

E Have the children compare the duration required for a penny to drop to the floor with the duration of a single interval between regularly repeated events. A metronome, clicking timer, or light flasher would be ideal, but a child clapping his hands is regular enough.

SHOW US HOW YOU CAN CLAP YOUR HANDS SO THAT THERE IS THE SAME DURATION BETWEEN CLAPS.

Suggest making the duration between claps longer and choose a child who claps about twice a second -- (100 to 120 times per minute.)

IF WE DROP THE PENNY WHEN JOHNNY CLAPS, WILL IT HIT THE FLOOR BEFORE HIS NEXT CLAP, AFTER THE CLAP, OR AT ABOUT THE SAME TIME?

Start the child clapping regularly, and give a signal such as "Ready, set, go," for dropping the penny. Do not let the clapping child see the penny being dropped because this will unconsciously affect his rhythm.

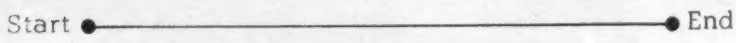
WAS THE PENNY DROPPED BEFORE, AFTER, OR ABOUT THE SAME TIME AS THE FIRST CLAP?

WHEN I SAID "GO," DID THE PENNY HIT THE FLOOR BEFORE OR AFTER THE NEXT CLAP?

If the children have difficulty telling whether the penny hit before or after the ensuing clap, merely point out that this is difficult because this duration is so very short.

Lesson 25: DURATION OF EVENTS THAT START TOGETHER

This lesson gives the children an opportunity to compare durations of events that have considerable variability, and to note the effect of practice on the performance of a task.



MATERIALS

-- for each two children --

- 2 clear plastic containers
- 2 sugar cubes
- craft or Popsicle stick for stirring
- 2 Fizzies

-- for the class --

- 2 mazes, to be traced by a child's finger, as large as feasible. These can be drawn on the chalkboard or on poster board.
- windup toys, such as cars, animals or play TV sets. Try to include two that take about the same to run down. The children may be able to provide some of the toys.

PROCEDURE

A Show water in a plastic cup and hold up a sugar cube.

WHAT DO YOU THINK WILL HAPPEN IF A SUGAR CUBE IS PUT IN WATER?

Drop the cube in and have the children watch. Repeat, but this time have a child stir the water without poking the cube. Have another child determine when the sugar has all dissolved ("has disappeared," "is all gone").

WHICH SUGAR CUBE TOOK THE GREATER DURATION OF TIME TO DISSOLVE?

HOW DO YOU KNOW?

HOW CAN YOU TEST TO MAKE SURE?

Give containers with water to each child, but a stirring stick to only one child of each pair. Distribute the sugar cubes to small groups and have them all start together. Children with sticks should use them to stir the water. The others should not disturb their solutions. Have each child signal when he can no longer see any sugar. Ask for observations of each pair:

WHAT HAPPENED IN YOUR GLASS?

WHICH CUBE SEEMS TO TAKE LONGER TO DISSOLVE?

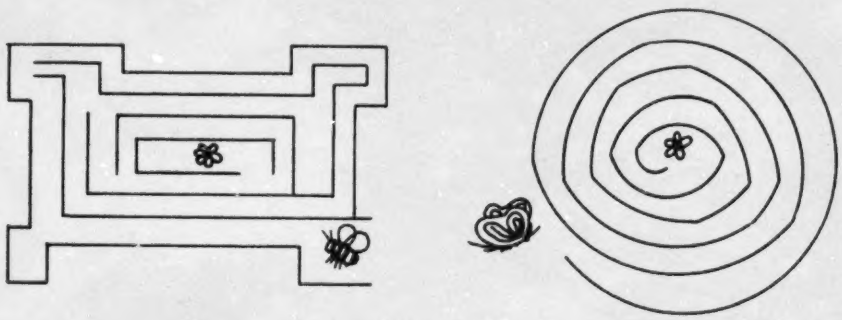
Other experiments may be done by pairs of children, if you find it feasible. You could give some children warm water, as well as stirring sticks. You could also substitute ice cubes for sugar cubes and have the children observe the durations of their melting.

[L] A popular repeat can be done with different sized cups -- 6, 7, or 8 oz. -- and Fizzies soft drink pellets. These rise to the top and can be easily observed. In addition to the time duration and time order questions, you might ask:

WHICH SEEMS TO TAKE LONGER, THE SUGAR OR THE FIZZIES?

Let the children make the comparison test.

C Draw on the chalkboard two finger mazes similar to the ones illustrated. One should be a coil maze with six or seven turns. The other should appear to be more difficult. However, with practice the latter will require a shorter duration to complete because of the shorter route.



START WITH FINGER AT THE BUTTERFLY OR BEE. GET TO THE FLOWER WITHOUT CROSSING ANY LINES.

Let two children trace the mazes at the same time. Have the rest of the class determine which takes the longer duration to trace. Have the children repeat the activity several times until practice alters the relative durations of the tracings.

D Start two windup toys at the same time and let them run down. Disregard the <u>distances</u> moved, as you want to separate the concept of duration from any notion of speed. This will be natural enough, as some of the windup toys do not move along the floor, and some do.

HOW CAN YOU TELL THAT THIS TOY, RAN FOR A GREATER DURATION THAN THIS ONE? (It is still going. It stopped last.)

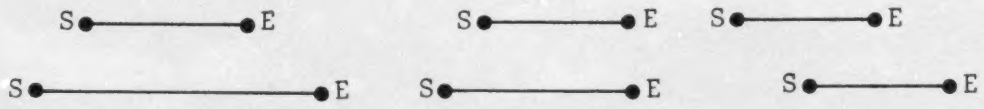
Windup cars can be used for repeated comparisons by having different children start them together.

E Strike a high note on the piano while depressing the hold pedal. A short time later strike a low note, still depressing the hold pedal. Release the pedal, which will cause both notes to cease.

WHICH NOTE HAD THE GREATER TIME DURATION? (The first note.)

Lesson 26: DURATION OF EVENTS THAT DO NOT START TOGETHER

Here you provide the children with experiences in comparing the durations of two events that do not have a common beginning. Events in this lesson have the following relations:



These relationships are not simple, and complete mastery by the children should not be expected. They should see that they can be sure of the comparisons only in the first two cases.

MATERIALS

- 2 or more windup toys from Lesson 25, or games or puzzles from other lessons of this unit (Activity A)

-- for Optional Activity B --

- record player
- 1 or more records selected by you for their suitability to use with various instruments such as your voice or a child's, a tambourine, tom-tom, drum or stick

or

- piano and accompaniment, other musicians in the class

PROCEDURE

[A] Show the children two windup toys.

HOW CAN WE TELL WHICH TOY TAKES THE LONGER DURATION TO RUN DOWN? (We could start both together and see which runs down first.)

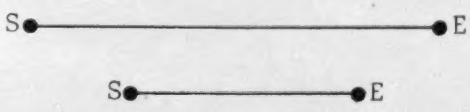
Suggest that sometimes it is difficult to start two toys at the same time.

IF THE TOYS ARE NOT STARTED TOGETHER, COULD WE STILL TELL WHICH HAS THE GREATER DURATION?

Perhaps many solutions will be offered. The children may suggest that the one which they expect to take longer be started first, and then the other, so that both finish together. Try this. If the two toys do not end together, ask:

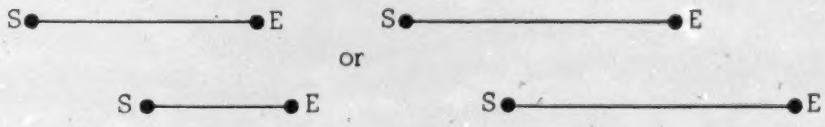
CAN WE STILL TELL WHICH TOY HAD THE GREATER DURATION? (We can if the toy that was started first stops last.)

A diagram of the situation could look like this:



Ask the children to suggest other events (finger mazes, writing on the board) where this same situation can be demonstrated.

The third situation is where the first event starts before the second and ends before the second. Diagramatically this appears as:



In this situation a comparison of the duration will appear simple when one duration is much longer than the other. Try to choose examples where the difference is obvious. Proceed then to examples where the difference is less obvious until there is some disagreement as to which event had the longer duration. Tell the class that you too are unable to detect which was longer.

B In this activity you may be able to use the talents of children in the class who have had some musical training. If not, you can use the piano or recordings and have a child accompany either, on signal from you, by beating a stick on a wooden block, beating a drum or tom-tom, or rattling a tambourine. Many combinations of two events are possible: you could play the piano and sing to your own playing for the three different time durations specified; you could have a child sing on signal; you could have children with two different instruments play them; or you could use both a record and a percussion instrument yourself.

Whatever combination of sounds you decide to use, make sure that the children put their heads down on their tables or cover their eyes. Be sure, too, that the sounds are easily distinguishable by the children. For example, you would not use a percussion instrument with a record that employed the same, or a very similar, sound.

Let's suppose, for the purpose of illustration, you have chosen to use a record and to sing along with it. You would always start your singing <u>after</u> the record has started playing, then end your singing at these three times: <u>before</u> the record ends, <u>at the same time as</u> the record ends, and <u>after</u> the record ends. (If children perform, tap each one when he is to start and when he is to stop.)

Hold a discussion about the time duration of the two events after each playing. If practicable, use several different records and a different accompaniment with each.

When using questions similar to those that follow, you might ask the children to bow their heads and indicate their choice by raising hands, in order to minimize the effect of group approval.

HOW MANY THINK MY SINGING HAD A LONGER TIME DURATION THAN THE RECORD?

HOW MANY THINK THE PLAYING OF THE RECORD HAD A LONGER TIME DURATION?

HOW MANY THINK I STARTED SINGING BEFORE THE RECORD STARTED?

HOW MANY THINK THE RECORD STARTED PLAYING BEFORE I STARTED SINGING?

HOW MANY THINK THE RECORD ENDED BEFORE MY SINGING DID?

HOW MANY THINK MY SINGING ENDED BEFORE THE RECORD STOPPED PLAYING?

HOW MANY THINK MY SINGING HAD A LONGER TIME DURATION THAN THE RECORD?

HOW MANY THINK THE RECORD HAD A LONGER TIME DURATION THAN MY SINGING?

Also to be discussed:

WHICH EVENT STARTED FIRST? WHICH STARTED LAST?

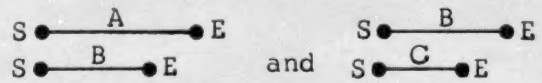
WHICH EVENT ENDED FIRST? WHICH EVENT ENDED LAST?

WHICH EVENTS CAN WE BE SURE HAD A LONGER DURATION? (Those that started before and ended after the second event. Those that started before, but ended at the same time as the second event.)

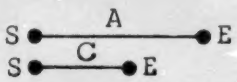
WHICH TIME DURATIONS CAN WE NOT COMPARE? (Those where the second event started after the first event and ended aft[illegible])

Lesson 27: INFERENCES FOR TIME DURATIONS

Here the children will infer comparisons in the context of time durations of three events. They will observe:



and will infer:



MATERIALS

- 2 plastic containers
- 2 Fizzies
- 1 sugar cube
- 1 spiral maze

PROCEDURE

Place the two containers in view of the children. Fill each one approximately 3/4 full of water. Drop the sugar cube and one of the Fizzies into their respective cups at the same time. Ask the children to observe.

> WHICH ONE USED THE GREATER TIME DURATION IN DISSOLVING? (The sugar.)
>
> HOW CAN YOU TELL? (They started together but the Fizzie melted first.)

Remove the sugar water from sight. Display the finger maze and select a child to run the maze. Have the child practice one or two times until he can run through the maze quite rapidly.

HOW CAN WE FIND OUT WHICH TAKES A LONGER DURATION -- A FIZZIE DISSOLVING OR THE CHILD RUNNING THE MAZE? (By starting both at the same time.)

Select another child to drop a Fizzie into the glass at your signal, "Go." Both events (running the maze and the dissolving of the Fizzie) should start together.

WHICH EVENT HAD THE LONGER DURATION? (Dissolving the fizzie.)

HOW DO YOU KNOW? (The girl running the maze was finished first.)

WHICH EVENT HAD THE GREATER DURATION -- RUNNING THE MAZE OR DISSOLVING THE SUGAR LUMP?

Allow the children time to discuss and defend their answers.

After this discussion, when the children have reached a decision about which event had the greater duration, let them check it by trying the sugar lump and maze again.